Pay Equality Among Heterogeneous Agents

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Abstract

A principal incentivizes a team of agents to work by choosing performance-contingent rewards. She desires to implement work by all agents as a unique Nash equilibrium. We develop a model in which agents are heterogeneous, both in their costs of effort and marginal contributions to team success. We identify necessary and sufficient conditions under which it is optimal to reward heterogeneous agents *equally*, and show that increasing inequality in the marginal productivity of agents can either increase or decrease pay inequality. Our results rationalize observed patterns of performance pay in many labor market settings, including professional sports leagues and the military.

Keywords: teams, incentives, unique implementation

JEL codes: D82, D86, D33

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1 Introduction

Research in labor economics has identified mechanisms that explain wage differentials among homogeneous workers, as well as the impacts of worker and firm heterogeneity on compensation structures (Krueger and Summers 1988, Postel-Vinay and Robin 2002, Farber, Herbst, Kuziemko, and Naidu 2021). Notably, many industries featuring wide distributions of agent productivity have surprisingly homogeneous pay structures.

Consider the National Basketball Association (NBA).[1] Compensation for NBA players is two-fold: annual "base pay" for the regular season and "performance pay" related to advancement in the playoffs. Using data from the 2021-22 NBA season,[2]the left panel of Figure 1 depicts the median, inter-quartile range, minimum, and maximum of player productivity by rank-order of players from most to least productive on each team using the Value Over Replacement Player (VORP) metric.[3] The middle (right) panel provides these statistics for annual base (performance) pay using the same rankings. Intuitively, annual base pay is highly positively correlated with player productivity. Counter-intuitively, however, performance pay is identical across players.[4] This pay structure holds across many other professional sports leagues and in other contexts, such as the military. For instance, Glaser and Rahman (2011) observe that in each branch of the United States Military, pay is fixed within a given rank, despite a wide distribution of within-rank service member productivity.

This paper proposes a theory to characterize performance pay in such settings. We consider a model in which a principal chooses performance pay to incentivize work by each agent in a team. Agents are heterogeneous in their cost of effort and marginal contribution to team success. The principal demands that work by all agents is the unique Nash equilibrium of the game induced by her chosen contract. Subject to this constraint, she minimizes the sum of transfers to the agents.

Previous literature has established that it is optimal to pay homogeneous agents

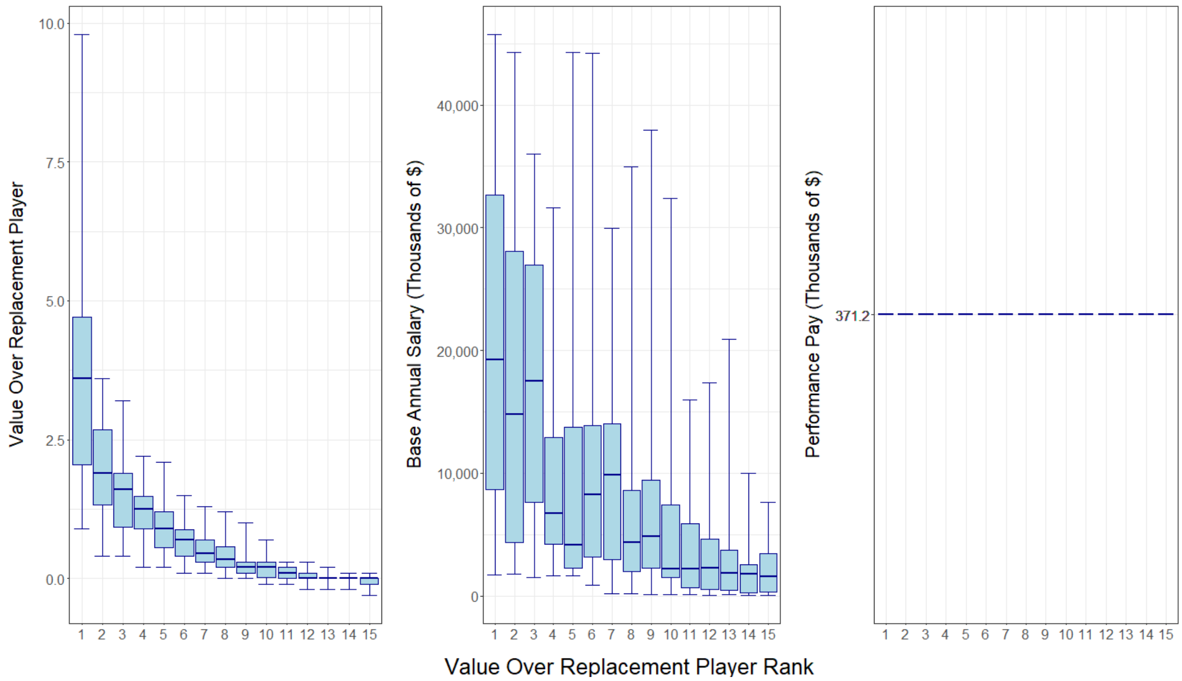
[1]The NBA features 30 teams, evenly split across 2 conferences, where each team plays an 82-game regular season with a roster of 15 players. The top 8 teams in each conference advance to the playoffs, where teams compete under a standard tournament bracket format.

[2]Productivity data is taken from `Basketball Reference`, base pay data is taken from `HoopsHype`, and performance pay data is taken from `TheSportsEconomist.com.`

[3]VORP measures the number of points per 100 possessions a player adds to his team compared to a replacement player (-2.0).

[4]During the 2021-22 season, each player on the championship-winning team was awarded $371,233, despite their heterogeneous contributions.

Figure 1: Productivity (left), Base Pay (middle), and Performance Pay (right) for NBA Players During 2021-22 Season



unequally (Segal 1999, Segal 2003, Winter 2004). We contribute to this literature by fully characterizing the manner in which agents would need to be heterogeneous in order for equal pay to be optimal (Proposition 1). When the cost of effort is identical across agents, we find that marginal contributions must be sufficiently heterogeneous to rationalize equal pay (Corollary 1). Moreover, a calibrated version of this model provides a remarkably good fit to our NBA productivity data (Figure 2). Finally, we show that the comparative statics of pay inequality are nontrivial: Increasing inequality in marginal contributions can either increase or decrease pay inequality (Proposition 2). This ambiguity points to the fruitfulness of estimating the model empirically and conducting counterfactual analyses.

2 Model

A principal contracts with $i = 1, \ldots, n$ agents to complete a project. Each agent completes a single task, and each task must be completed for the project to succeed.

In any other case, the project fails. Each agent chooses an action, "work" or "shirk". If agent i works, then she completes her task with probability one and incurs an effort cost of $c_i > 0$. If agent i shirks, then she completes her task with probability $\alpha_i \in (0, 1)$ and incurs no effort cost. Hence, an agent's marginal contribution to project success, i.e., her productivity, is measured by $m_i := 1 - \alpha_i \in (0, 1)$. We assume, for simplicity, that agents are ordered in terms of productivity: $m_1 > ... > m_n$. Finally, all parties are risk-neutral and the agents are protected by limited liability so that they cannot receive negative wages.

The principal can only observe the overall success or failure of the project. Hence, she chooses a **contract** $v \in \mathbb{R}^n_+$, where $v_i \geq 0$ corresponds to the reward agent i receives conditional on project success. If the project fails, all agents receive a reward of zero. A contract $v \in \mathbb{R}^n_+$ that uniquely implements work by all agents as the unique Nash equilibrium of the game induced by the contract is called an **incentive inducing (INI) mechanism**. The principal's problem is to choose an INI mechanism that minimizes the sum of payments to the agents. A solution to the principal's problem is called an **optimal INI mechanism**.

We make one technically-motivated assumption. Because the set of contracts inducing work by all agents as a unique Nash equilibrium is open, there need not exist an optimal INI mechanism. To ensure existence, we assume that each agent works when indifferent between working and shirking, given their conjecture about the behavior of the other agents. (An equivalent approach in the literature is to instead define an optimal INI mechanism as one in which (i) no INI mechanism results in a lower sum of rewards and (ii) for any $\epsilon > 0$, $(v_i + \epsilon)_i$ is an INI mechanism.)

We remark that our model reduces to the model of Winter (2004) when $c_i = c_j$ and $\alpha_i = \alpha_j$ for all i and j. Like us, Halac, Kremer, and Winter (2022) consider the case in which costs of effort can differ across agents. However, their main analysis focuses on the case in which the probability of task completion conditional on shirking is homogeneous across agents, i.e., $\alpha_i = \alpha_j$ for all i and j, and the implications of cost heterogeneity on optimal monitoring. Our model relaxes this assumption in order to analyze the relationship between heterogeneity in productivity and pay.[5]

[5]See, also, Gueye, Quérou, and Soubeyran (2022) who study the role of heterogeneity in inequality aversion on inequality of performance pay.

3 Optimal INI Mechanisms

We now characterize optimal INI mechanisms.

Proposition 1. *The contract v is an optimal INI mechanism if and only if there exists a permutation of agents $\pi : \{1, ..., n\} \to \{1, ..., n\}$ satisfying*

$$\pi(i) < \pi(j) \quad \textit{only if} \quad \frac{c_i}{c_j} \leq \bar{c}(m_i, m_j) := \left(\frac{m_i^2}{1 - m_i}\right)\left(\frac{1 - m_j}{m_j^2}\right) \tag{1}$$

and for which

$$v_i = \left(\frac{c_i}{m_i}\right)\left(\frac{1}{\prod_{\{j:\pi(j)>\pi(i)\}}(1 - m_j)}\right).$$

Proof. A standard argument in the literature establishes that if the contract v is an INI mechanism, then there exists a permutation of agents $\pi : \{1, ..., n\} \to \{1, ..., n\}$ such that, for any $i \in \{1, ..., n\}$, agent i prefers to work in any strategy profile in which agents $\{k : \pi(k) < \pi(i)\}$ work. In addition, in any *optimal* INI mechanism, agent i is indifferent between working and shirking in any strategy profile in which agents $\{k : \pi(k) < \pi(i)\}$ work and all others shirk. The binding incentive constraints yield the equations for reward pay.

Novel to our setting, we show that an optimal permutation, π, must satisfy (1). Towards contradiction and without loss of generality, suppose that $\pi(i) < \pi(j)$ for agents i and $j = i + 1$, and that (1) does not hold. We claim that the permutation π' with $\pi'(i) = \pi(j)$ and $\pi'(j) = \pi(i)$ that is otherwise identical to π strictly reduces reward payments when agents are paid optimally. It suffices to show that

$$\left(\frac{c_i}{m_i}\right)\left(\frac{1}{1 - m_j}\right) + \left(\frac{c_j}{m_j}\right) > \left(\frac{c_i}{m_i}\right) + \left(\frac{c_j}{m_j}\right)\left(\frac{1}{1 - m_i}\right)$$

because optimal reward pay for agents below $\min\{i, j\}$ and above $\max\{i, j\}$ is identical across permutations. Re-arranging the inequality yields (1). Hence, the original permutation of agents could not have been optimal.

To prove that the conditions in the Proposition are sufficient, notice that the only way to decrease $\sum_i v_i$ is to strictly decrease v_i for some i. Call such a vector v', order agents by the permutation corresponding to the original INI mechanism, and let k be the largest index for which $v'_k < v_k$. Then, there exists a Nash equilibrium in which players $k, ..., n$ shirk and all others work. So, any contract with a lower sum of reward payments could not have been an INI mechanism. $\square$

Proposition 1 establishes a wide range of conditions under which equal pay is optimal for unequal agents. To gain intuition, it is instructive to consider the case in which agents have an identical effort cost, a plausible approximation in professional sports leagues and in the military.

Corollary 1. *Suppose $c_1 = ... = c_n$. Then, the identity permutation is the unique optimal permutation. In addition, $v_1 = ... = v_n$ in an optimal INI mechanism if and only if*

$$m_i = \frac{m_{i-1}}{1 + m_{i-1}} \quad \textit{for } i = 2, ..., n. \tag{2}$$

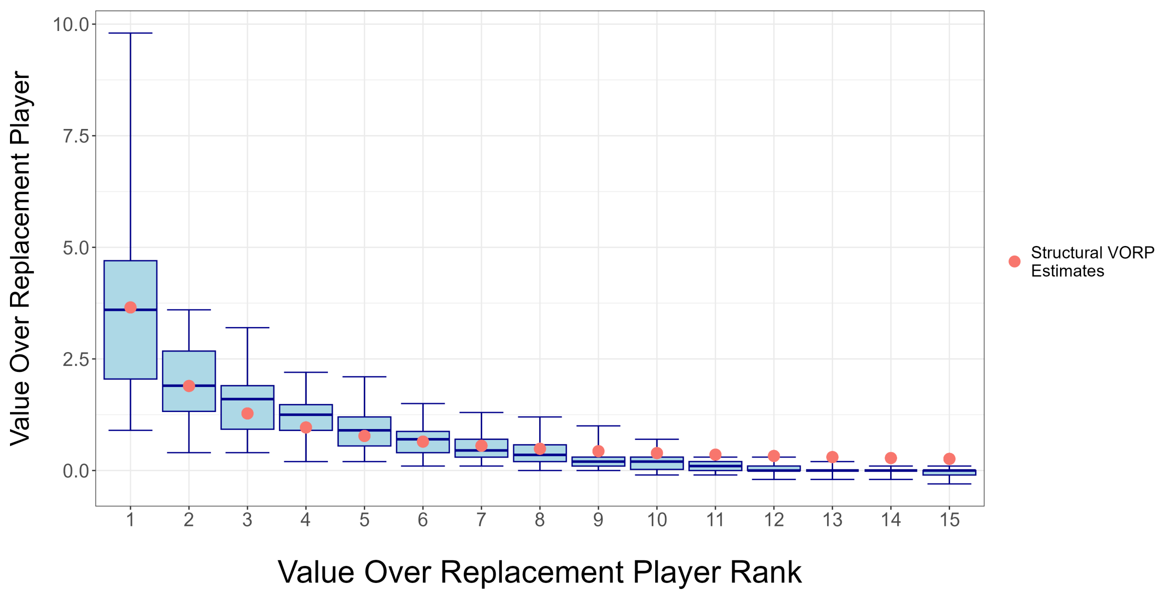
Corollary 1 establishes that rationalizing identical pay across agents necessitates dispersion in their productivities. This is in stark contrast to the standard principal-agent model in which the principal requires only that working by all agents is one of possibly many Nash equilibria. Specifically, the standard model predicts that all agents must have identical marginal productivities if they have the same cost of effort (agent i's optimal reward is $\frac{c_i}{m_i}$).

The structural properties of our model enable us to well-approximate the distribution of marginal productivities in the NBA without resorting to heterogeneity in effort costs. To calibrate the model to the data, we solve the following optimization problem:

$$\min_{z \in \mathbb{R}, m_1 \in [0,1]} \quad \frac{1}{15} \sum_{i=1}^{15} (z * m_i - d_i)^2$$
$$\text{subject to}$$
$$m_i = \frac{m_{i-1}}{1 + m_{i-1}} \quad \text{for } i = 2, ..., 15,$$

where z is the return to a unit increase in m_i in terms of the VORP index and d_i is the median VORP of the i-th ranked player in the data. The optimal values are $(m_1^*, z^*) \approx (0.93, 3.93)$ leading to a mean-squared error of approximately 0.05. The red dots in Figure 2 plot the model's predicted VORP values, $(z^* m_i^*)_i$, where the values of m_i^* for $i = 2, ..., n$ are pinned down by the structural equal-pay constraints, against the empirical distributions of VORP values.

Figure 2: Observed vs. Structural VORP Values



4 Comparative Statics of Pay Inequality

We conclude by establishing some counter-intuitive results about the relationship between inequality in productivity and inequality of pay.

4.1 Two agents

We provide an essentially complete characterization in the case in which there are two agents. Let π be the unique optimal permutation of agents and v^π be the optimal vector of reward pay. We say that *a local increase in inequality of productivity leads to a local increase (decrease) in inequality of reward pay* if

$$\frac{\partial(|v_1^\pi - v_2^\pi|)}{\partial m_1} > \ (<)\ 0 \quad \text{and} \quad \frac{\partial(|v_1^\pi - v_2^\pi|)}{\partial m_2} < \ (>)\ 0.$$

That is, increasing the productivity of the most productive agent and decreasing the productivity of the least productive agent strictly increases (decreases) their absolute difference in reward pay.

Proposition 2. *Suppose $n = 2$ and $\frac{c_1}{c_2} < \bar{c}(m_1, m_2)$ so that the identity permutation* *is optimal.*[6] *There exists a value* $\hat{c} \in (0, \bar{c}(m_1, m_2))$ *such that the following properties hold:*

1. *If* $\frac{c_1}{c_2} > \hat{c}$, *then a local increase in inequality of productivity leads to a local decrease in inequality of reward pay.*

2. *If* $\frac{c_1}{c_2} < \hat{c}$, *then a local increase in inequality of productivity leads to a local increase in inequality of reward pay.*

Proof. By Proposition 1, the optimal reward pay vector sets

$$v_1^\pi = \left(\frac{c_1}{m_1}\right)\left(\frac{1}{1-m_2}\right) \quad \text{and} \quad v_2^\pi = \frac{c_2}{m_2}.$$

If $\frac{c_1}{c_2} > \hat{c} := m_1\left(\frac{1-m_2}{m_2}\right)$, then $v_1^\pi > v_2^\pi$. Notice that v_1^π is decreasing in m_1 and v_2^π is constant in m_1. Hence, $\frac{\partial(|v_1^\pi - v_2^\pi|)}{\partial m_1} < 0$ because $v_1^\pi > v_2^\pi$. On the other hand, increasing m_2 increases v_1^π and decreases v_2^π. So, $\frac{\partial(|v_1^\pi - v_2^\pi|)}{\partial m_2} > 0$. The result for the case in which $\frac{c_1}{c_2} < \hat{c}$ follows from the previous steps, switching signs when necessary. □

The economic forces behind Proposition 2 are as follows: When the more productive agent becomes even more productive, she needs to be compensated less for project success because her effort has a stronger effect on the probability with which she receives a reward. On the other hand, a decrease in the marginal influence of agent 2 on project success has two effects. First, it depresses agent 1's performance pay by reducing the risk she incurs when agent 2 shirks. Second, it *increases* the reward pay required to incentivize agent 2 to exert effort; her return to effort is diminished, requiring her to be paid more in order for her to optimally exert the same amount of effort. Whether or not these adjustments increase or decrease pay inequality depend on the order between v_1 and v_2 under the initial parameters. If $\frac{c_1}{c_2}$ is sufficiently large, then $v_1 > v_2$ and a local increase in productivity dispersion leads to a local contraction in $|v_1 - v_2|$. The opposite occurs when $\frac{c_1}{c_2}$ is sufficiently small.

4.2 More than two agents

Our characterization in Proposition 2 extends to the case of $n > 2$ agents when considering pay inequality between adjacent agents in an optimal permutation. However,

[6] Analogous results hold when this inequality is flipped.

obtaining a clean condition on the primitives of the model that govern comparative statics on the entire distribution of productivities is complicated for two reasons. First, dispersion in the productivities of nonadjacent agents results in competing effects on the pay of those between them. Second, it is no longer clear how to measure inequality in the distributions of marginal productivity vectors and reward pay vectors. A natural direction for future research would be to define an order on distributions of productivity so that if one distribution is larger than another in this order, then inequality in reward pay is correspondingly smaller.

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