Pay Equality Among Heterogeneous Agents^{*} Ryan Hoffman[†] Ashwin Kambhampati[‡] Scott Kaplan[§] April 7, 2023

Abstract

A principal incentivizes a team of agents to work by choosing performancecontingent rewards. She desires to implement work by all agents as a unique Nash equilibrium. We identify necessary and sufficient conditions under which it is optimal to reward heterogeneous agents *equally*, and show that increasing inequality in the marginal productivities of agents can either increase or decrease pay inequality. Our results rationalize patterns of performance pay in many labor market settings, including professional sports leagues and the military.

Keywords: teams, incentives, unique implementation, inequality

JEL codes: D33, D82, D86

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1 Introduction

Research in labor economics has identified mechanisms that explain wage differentials among homogeneous workers, as well as the impacts of worker and firm heterogeneity on compensation structures (Krueger and Summers 1988, Postel-Vinay and Robin 2002, Farber, Herbst, Kuziemko, and Naidu 2021). Notably, many industries featuring relatively large dispersion in agent productivity have surprisingly homogeneous pay structures.

Consider the National Basketball Association (NBA).¹ Compensation for NBA players is two-fold: annual "base pay" for the regular season and "performance pay" related to advancement in the playoffs. Using data from the 2021-22 NBA season,²the left panel of Figure 1 depicts the median, inter-quartile range, minimum, and maximum of player productivity by rank-order of players from most to least productive on each team using the Value Over Replacement Player (VORP) metric.³ The middle (right) panel provides these statistics for annual base (performance) pay using the same rankings. Intuitively, annual base pay is highly positively correlated with player productivity. Counter-intuitively, performance pay is identical across players.⁴ This pay structure holds across many other professional sports leagues and in other contexts, such as the military. For instance, Glaser and Rahman (2011) observe that in each branch of the United States Military, pay is fixed within a given rank, despite a wide distribution of within-rank service member productivity.

This paper proposes a theory to characterize performance pay in such settings. Following Winter (2004), we consider a model in which a principal chooses performance pay to incentivize "work" rather than "shirk" by each agent in a team. The team succeeds if and only if each agent successfully completes their task. As in Winter (2004), work ensures success. In contrast, the probability of success conditional on shirking differs across agents. The principal demands that work by all agents is the unique Nash equilibrium of the game induced by her chosen contract. Subject to

 $^{^{1}}$ The NBA features 30 teams, evenly split across 2 conferences, and each team has a roster of 15 players. The top 8 teams in each conference advance to the playoffs and compete under a standard tournament bracket format.

²Productivity data is taken from Basketball Reference, base pay data is taken from HoopsHype, and performance pay data is taken from TheSportsEconomist.com.

 $^{^{3}}$ VORP measures the number of points per 100 possessions a player adds to his team compared to a replacement player (-2.0).

⁴During the 2021-22 season, each player on the championship-winning team was awarded \$371,233, despite their heterogeneous contributions.

Figure 1: Productivity (left), Base Pay (middle), and Performance Pay (right) for NBA Players During 2021-22 Season



this constraint, she minimizes the sum of transfers to the agents.

Previous literature has established that it is optimal to pay agents with identical marginal contributions unequally (Segal 1999, Segal 2003, Winter 2004). We contribute to this literature by characterizing the manner in which agents would need to be heterogeneous in order for equal pay to be optimal (Proposition 1). As a simple illustration of these conditions, suppose that there are two agents, each with a cost of effort of 1. Agent 1 succeeds with probability 1/2 if she shirks, whereas agent 2 succeeds with probability 2/3 if she shirks. By paying each a bonus of 3, the principal ensures team success at minimal cost. Agent 1 is willing to work even if agent 2 shirks. Agent 2, knowing that agent 1 will work, is willing to work. Notice that the marginal productivity of agent 1 is $m_1 := 1 - 1/2$, while the marginal productivity of agent 2 is $m_2 := 1 - 2/3$. Therefore,

$$m_2 = \frac{m_1}{1+m_1}.$$

With more than two agents, we show that if this relationship holds between each agent

and the next most marginally productive agent, then equal bonuses are optimal.

Because rationalizing equal pay necessitates sufficient dispersion in marginal productivities, a calibrated version of our model provides a remarkably good fit to our NBA productivity data (Figure 2). Nevertheless, the comparative statics of pay inequality are nontrivial: Increasing inequality in marginal contributions can either increase or decrease pay inequality (Proposition 2). This ambiguity points to the potential fruitfulness of estimating the model empirically and conducting counterfactual analyses.

2 Model

A principal contracts with i = 1, ..., n agents to complete a project. Each agent completes a single task, and each task must be completed for the project to succeed (otherwise, it fails). Each agent chooses an action, "work" or "shirk". If agent *i* works, then she completes her task with probability one and incurs an effort cost of c > 0.5 If agent *i* shirks, then she completes her task with probability $\alpha_i \in (0, 1)$ and incurs no effort cost. Hence, an agent's marginal contribution to project success, i.e., her productivity, is measured by $m_i := 1 - \alpha_i \in (0, 1)$. We assume, for simplicity, that agents are ordered in terms of productivity: $m_1 > ... > m_n$. Finally, all parties are risk-neutral and the agents are protected by limited liability so that they cannot receive negative wages.

The principal can only observe the overall success or failure of the project. Hence, she chooses a **contract** $v \in \mathbb{R}^n_+$, where $v_i \geq 0$ corresponds to the reward agent *i* receives conditional on project success. If the project fails, all agents receive a reward of zero. A contract $v \in \mathbb{R}^n_+$ that uniquely implements work by all agents as the unique Nash equilibrium of the game induced by the contract is called an **incentive inducing (INI) mechanism**. The principal's problem is to choose an INI mechanism that minimizes the sum of payments to the agents. A solution to the principal's problem is called an **optimal INI mechanism**.

We make one technically-motivated assumption. Because the set of contracts inducing work by all agents as a unique Nash equilibrium is open, there need not exist an optimal INI mechanism. To ensure existence, we assume that each agent

⁵We incorporate heterogeneous effort costs in Section 5.

works when indifferent between working and shirking, given their conjecture about the behavior of the other agents.⁶

We remark that our model reduces to the model of Winter (2004) when $\alpha_i = \alpha_j$ for all *i* and *j*. Halac, Kremer, and Winter (2022) focus on the case in which costs of effort differ across agents, but marginal contributions are constant, and study the implications of cost heterogeneity on optimal monitoring. Our model relaxes the constant marginal contribution assumption in order to analyze the relationship between heterogeneity in productivity and homogeneity in pay.⁷

3 Optimal INI Mechanisms

We now characterize optimal INI mechanisms.

Proposition 1. The contract v is an optimal INI mechanism if and only if

$$v_i = \left(\frac{c}{m_i}\right) \left(\frac{1}{\prod_{j=i+1}^n (1-m_j)}\right).$$

Hence, $v_1 = ... = v_n$ in an optimal INI mechanism if and only if

$$m_i = \frac{m_{i-1}}{1+m_{i-1}}$$
 for $i = 2, ..., n$.

Proof. A standard argument in the literature establishes that if the contract v is an INI mechanism, then there exists a permutation of agents $\pi : \{1, ..., n\} \rightarrow \{1, ..., n\}$ such that, for any $i \in \{1, ..., n\}$, agent i prefers to work in any strategy profile in which agents $\{k : \pi(k) < \pi(i)\}$ work. In addition, in any optimal INI mechanism, agent i is indifferent between working and shirking in any strategy profile in which agents $\{k : \pi(k) < \pi(i)\}$ work and all others shirk. The binding incentive constraints imply that optimal reward pay is given by

$$v_i = \left(\frac{c}{m_i}\right) \left(\frac{1}{\prod_{\{j:\pi(j)>\pi(i)\}}(1-m_j)}\right).$$

⁶An equivalent approach in the literature is to instead define an optimal INI mechanism as one in which (i) no INI mechanism results in a lower sum of rewards and (ii) for any $\epsilon > 0$, $(v_i + \epsilon)_i$ is an INI mechanism.

⁷See, also, Gueye, Quérou, and Soubeyran (2022) who study the role of inequality aversion on inequality of performance pay.

We now show that any optimal permutation must have $\pi(i) = i$ for all *i*. Towards contradiction and without loss of generality, suppose that $\pi(i) < \pi(j)$ for agents *i* and j = i + 1. We claim that the permutation π' with $\pi'(i) = \pi(j)$ and $\pi'(j) = \pi(i)$ that is otherwise identical to π strictly reduces reward payments when agents are paid optimally. It suffices to show that

$$\left(\frac{c}{m_i}\right) \left(\frac{1}{1-m_j}\right) + \left(\frac{c}{m_j}\right) > \left(\frac{c}{m_i}\right) + \left(\frac{c}{m_j}\right) \left(\frac{1}{1-m_i}\right)$$
$$\iff \left(\frac{m_j^2}{1-m_j}\right) > \left(\frac{m_i^2}{1-m_i}\right)$$

because optimal reward pay for agents below $\min\{i, j\}$ and above $\max\{i, j\}$ is unchanged. The final inequality holds because $m_j > m_i$ (recall, $m_1 < \dots < m_n$). Hence, the original permutation of agents could not have been optimal.

To prove that the conditions in the Proposition are sufficient, notice that the only way to decrease $\sum_i v_i$ is to strictly decrease v_i for some *i*. Call such a vector v' and let *k* be the smallest index for which $v'_k < v_k$. Then, there exists a Nash equilibrium in which players k, ..., n shirk and all others work. So, any contract with a lower sum of reward payments could not have been an INI mechanism.

Proposition 1 establishes that rationalizing identical pay across agents necessitates dispersion in their productivities, in contrast to the prediction under partial implementation. Specifically, if the principal requires only that working by all agents is one of possibly many Nash equilibria, then agent *i*'s optimal reward is $\frac{c}{m_i}$. So, all agents must have the same marginal productivity to have the same reward pay.

The structural properties of our model enable us to well-approximate the distribution of marginal productivities in the NBA without resorting to heterogeneity in effort costs. To calibrate the model to the data, we solve the following optimization problem:

$$\min_{z \in \mathbb{R}, m_1 \in [0,1]} \quad \frac{1}{15} \sum_{i=1}^{15} (z * m_i - d_i)^2$$
subject to
$$m_i = \frac{m_{i-1}}{1 + m_{i-1}} \quad \text{for } i = 2, ..., 15,$$

where z is a scalar representing the return to a unit increase in m_i in terms of the



Figure 2: Observed vs. Structural VORP Values

VORP index and d_i is the median VORP of the *i*-th ranked player in the data. The optimal values are $(m_1^*, z^*) \approx (0.93, 3.93)$ leading to a mean-squared error of approximately 0.05. The red dots in Figure 2 plot the model's predicted VORP values, $(z^*m_i^*)_i$, where the values of m_i^* for i = 2, ..., n are pinned down by the structural equal-pay constraints, against the empirical distributions of VORP values.

4 Comparative Statics of Pay Inequality

We conclude by establishing some counter-intuitive results about the relationship between inequality in productivity and inequality of pay.

4.1 Two agents

We provide an essentially complete characterization in the case in which there are two agents. Given an optimal vector of reward pay v, say that a local increase in inequality of productivity leads to a local increase (decrease) in inequality of reward pay if

$$\frac{\partial(|v_1 - v_2|)}{\partial m_1} > \ (<) \ 0 \quad \text{and} \quad \frac{\partial(|v_1 - v_2|)}{\partial m_2} < \ (>) \ 0.$$

That is, increasing the productivity of the most productive agent and decreasing the productivity of the least productive agent strictly increases (decreases) their absolute difference in reward pay.

Proposition 2. If n = 2, then the following properties hold:

- 1. If $m_1 < \frac{m_2}{1-m_2}$, then a local increase in inequality of productivity leads to a local decrease in inequality of reward pay.
- 2. If $m_1 > \frac{m_2}{1-m_2}$, then a local increase in inequality of productivity leads to a local increase in inequality of reward pay.

Proof. By Proposition 1, the optimal reward pay vector sets

$$v_1 = \left(\frac{c}{m_1}\right) \left(\frac{1}{1-m_2}\right) \quad \text{and} \quad v_2 = \frac{c}{m_2}.$$

If $m_1 < \frac{m_2}{1-m_2}$, then $v_1 > v_2$. Notice that v_1 is decreasing in m_1 and v_2 is constant in m_1 . Hence, $\frac{\partial(|v_1-v_2|)}{\partial m_1} < 0$ because $v_1 > v_2$. On the other hand, increasing m_2 increases v_1 and decreases v_2 . So, $\frac{\partial(|v_1-v_2|)}{\partial m_2} > 0$. The result for the case in which $m_1 > \frac{m_2}{1-m_2}$ follows from analogous steps.

The economic forces behind Proposition 2 are as follows: When the more productive agent becomes even more productive, she needs to be compensated less for project success because her effort has a stronger effect on the probability with which she receives a reward. On the other hand, a decrease in the marginal influence of agent 2 on project success has two effects. First, it depresses agent 1's performance pay by reducing the risk she incurs when agent 2 shirks. Second, it *increases* the reward pay required to incentivize agent 2 to exert effort; her return to effort is diminished, requiring her to be paid more in order for her to optimally exert the same amount of effort. Whether or not these adjustments increase or decrease pay inequality depend on the order between v_1 and v_2 under the initial parameters. If m_1 is sufficiently small, then $v_1 > v_2$ and a local increase in productivity dispersion leads to a local contraction in $|v_1 - v_2|$. The opposite occurs when m_1 is sufficiently large.

4.2 More than two agents

Our characterization in Proposition 2 extends to the case of n > 2 agents when considering pay inequality between adjacent agents. However, obtaining a clean condition on the primitives of the model that govern comparative statics on the entire distribution of productivities is complicated for two reasons. First, dispersion in the productivities of nonadjacent agents results in competing effects on the pay of those between them. Second, it is no longer clear how to measure inequality in the distributions of marginal productivity vectors and reward pay vectors.

5 Heterogeneous Effort Costs

We conclude by discussing how our analysis extends when agents also have heterogeneous costs of effort. A simple modification of the proof of Proposition 1 establishes that the contract v is an optimal INI mechanism if and only if there exists a permutation of agents $\pi : \{1, ..., n\} \rightarrow \{1, ..., n\}$ satisfying

$$\pi(i) < \pi(j) \quad \text{only if} \quad \frac{c_i}{c_j} \le \bar{c}(m_i, m_j) := \left(\frac{m_i^2}{1 - m_i}\right) \left(\frac{1 - m_j}{m_j^2}\right)$$

and for which

$$v_i = \left(\frac{c_i}{m_i}\right) \left(\frac{1}{\prod_{\{j:\pi(j)>\pi(i)\}}(1-m_j)}\right),\,$$

where c_i is agent *i*'s cost of effort. Identifying conditions under which equal pay is optimal thus requires consideration of the manner in which changes in marginal productivities affect the optimal permutation of agents.

Nevertheless, the arguments in Proposition 2 can still be extended. We obtain the following result: Suppose n = 2 and $\frac{c_1}{c_2} < \bar{c}(m_1, m_2)$ so that the identity permutation is optimal.⁸ Then, letting $\hat{c} := m_1 \left(\frac{1-m_2}{m_2}\right) < \bar{c}(m_1, m_2)$, the following properties hold:

- 1. If $\frac{c_1}{c_2} > \hat{c}$, then a local increase in inequality of productivity leads to a local decrease in inequality of reward pay.
- 2. If $\frac{c_1}{c_2} < \hat{c}$, then a local increase in inequality of productivity leads to a local increase in inequality of reward pay.

The relationship between $\frac{c_1}{c_2}$ and the threshold \hat{c} determines whether $v_1 > v_2$ or $v_1 < v_2$, which is once again crucial in determining the effect of increasing inequality

⁸Similar results hold when this inequality is flipped.

in productivity on inequality of pay.

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