Why Informationally Diverse Teams Need Not Form, Even When Efficient *

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Abstract

We introduce a model of team formation in which workers first match and then produce correlated signals about an unknown state. While it is efficient to maximize the number of informationally diverse teams, such teams need not form in equilibrium when output is shared equally. Our analysis identifies the two sources of matching inefficiency: (i) workers may form diverse teams that are beneficial to its members, but force excluded workers to form homogeneous teams, and (ii) even when a diverse team is efficient, a worker may prefer to join a homogeneous team if she can exert less effort than her teammate. We completely characterize each inefficiency.

Keywords: Matching, Teams, Information Acquisition, Correlation

JEL Classification: C78, L23, D83

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1 Introduction

1.1 Background

Self-organized teams are playing an increasingly important role in economic activity. From 1987 to 1996, the fraction of Fortune 1000 firms with workers in self-managed work teams rose from 27 percent to 78 percent (Lawler, Mohrman and Benson (2001) and Lazear and Shaw (2007)). More recently, a 2016 survey of more than 7,000 executives in over 130 countries indicates that organizations are increasingly operating as a network of teams in which workers engage in self-directed research (Deloitte, 2016). These human resources trends are particularly important in organizations such as Universities (Wuchty, Jones and Uzzi (2007)) and large technology companies, like Google and Amazon, that rely on flexible internal labor markets in order to take advantage of informational complementarities among workers with diverse backgrounds. Yet while the free-ridership problem within teams has garnered considerable theoretical attention (see, for instance, Holmström (1982), Legros and Matthews (1993), and Winter (2004)), less has been devoted to the study of how moral hazard within teams affects matching. Furthermore, little existing work studies this interaction in the context of the production of information.[1]

The case of the Danish hearing-aid manufacturer Oticon illustrates well these broad trends in research and development, as well as the incentive problems that arise when decision making is delegated to productive actors themselves (see Foss (2003) for a comprehensive account). In 1987, Oticon lost almost half of its equity when its competitors began selling cosmetically superior devices. In an attempt to regain its competitive advantage, Oticon re-structured its research department, replacing vertical, hierarchical production with horizontal, project-based team production (Foss (2003) coins this organizational form a *spaghetti organization*). Beyond cosmetic changes to the office spaces — desks were no longer permanent and were located in large open spaces — there was extensive delegation of decision rights. Most notably, employees chose which projects (teams) they would join and had discretion over their compensation.

[1]Subsequent to the first circulated draft of this paper, Kaya and Vereshchagina (2022) study the optimal sorting of workers to teams who differ in their ability to acquire information and engage in team production. As in Chade and Eeckhout (2018), they focus on the sorting of workers of differing expertise into teams, whereas we hold this dimension fixed and consider heterogeneity in the correlation of workers' information. Kambhampati and Segura-Rodriguez (2022) study the optimal allocation of workers to teams in a standard production setting in the presence of both moral hazard and adverse selection. They then identify when decentralized sorting is an optimal organizational structure.

At first, these organizational changes were profitable. Eliminating hierarchies and allowing workers to lead their own teams enabled the firm to take advantage of the existing information dispersed among its workers (Kao, 1996).[2] However, new problems arose. First, competition meant that "anybody [at a project] could leave at will, if noticing a superior opportunity in the internal job market" (Foss, 2003). Second, some teams were far better than others "in terms of how well the team members worked together and what the outcome of team effort was" (Larsen, 2002). These problems eventually led Oticon to introduce a company-wide employee stock option program and selectively intervene in the assignment of roles to workers within teams, designating particular workers as project managers.[3]

1.2 This Paper

We posit a model of moral hazard and matching in the context of information production to better understand the managerial problems faced by firms that decentralize information production and to rationalize management solutions observed within companies like Oticon. In the setting we study, workers form teams (match) in order to forecast the value of a Gaussian state. Each worker then acquires any number of costly Gaussian signals about it. Pairwise correlations between signals in a team can be positive, i.e., the team is *homogeneous*, or negative, i.e., the team is *diverse*. After observing all signals produced within a team, each team guesses the state and each worker receives a payoff proportional to the quadratic distance between her team's forecast and the state realization.

The matching environment features imperfectly transferable utility (Legros and Newman (2007)); one worker cannot compensate another for producing more or less signals than her and a team's profit is divided equally. The literature on partnerships (see, for

[2] Oticon's CEO commented that decentralization "improved markedly [Oticon's] ability to invent new ideas, concepts, and make use of what [Oticon] actually [had]" (Kao, 1996). In particular, the firm was able to revive old projects that later turned out to be profitable.

[3] While a prominent example, Oticon is not the only company to have experimented with decentralized research teams and had problems. In 2012, the multibillion-dollar video-game developer Valve publicly released a New Employer Handbook describing the company's non-hierarchical organizational structure. Valve's co-founder adopted this approach in the hope of spurring the company's research and innovation (Keighley, 2020). But, once again, decentralization led to new problems. First, talented workers refused to leave prestigious projects, and it became hard for other projects to recruit them. Second, the flat management model gave workers latitude to "minimize their work" because of the lack of "checks and balances" (Grey, 2013). In 2014, GitHub introduced a middle-management level to supervise its previously unsupervised allocation system of workers to teams (Rusli, 2014). More recently, in 2016, Medium abandoned its use of holocracy, a system "designed to move companies away from rigid corporate structures and toward decentralized management and dynamic composition" (Doyle, 2016).

instance, Farrell and Scotchmer (1988) and Sherstyuk (1998)) has argued that social convention and social norms might rule out unequal division of surplus even if transfers within teams are permitted. Indeed, academics often receive equal credit for joint work, even if work is not divided equally. As in Farrell and Scotchmer (1988), our interest is in identifying the social cost that equal-sharing rules play in generating utilitarian welfare losses when sorting is endogenous. Utilitarian welfare is the relevant efficiency notion because, if it is not maximized, then there is a clear way to improve the welfare of all workers even without interfering with the convention of equal-sharing. Specifically, the firm can simply re-assign workers and compensate them directly for the gain and/or loss accrued to them due to their change in teammate. The additional surplus generated by this change can either be distributed among the workers or be pocketed by management.

We identify and completely characterize the two channels leading to matching inefficiency. First, productive, diverse teams composed of workers producing negatively correlated signals may form at the expense of excluded workers who must form homogeneous teams whose workers produce positively correlated information. We call this phenomenon *stratification inefficiency*, which coheres with the observation that teams inside flat organizations tend to be unequal in productivity and with the existing literature on matching with nontransferabilities. Second, diverse teams may not form even when efficient; a worker in such a team may prefer to join a homogeneous team if, in this deviating team, she can exert less effort. We call this phenomenon *asymmetric effort inefficiency*, which rationalizes observations of unequal effort between employees in the same team and which, to our knowledge, has not been systematically studied in the literature.

1.3 Overview of Analysis

The formal analysis proceeds as follows. First, we consider a benchmark transferable utility environment in which workers sort into teams, coordinate on their signal-acquisition profile, and freely divide team surplus. We show that, for a homogeneous team, efficiency calls for only one of the two workers to acquire a strictly positive number of signals due to the redundancy of matched information. On the other hand, for a diverse team, any efficient signal-acquisition profile is symmetric (Proposition 1). When team surplus can be divided freely, there is an equivalence among efficient matchings, stable matchings, and those which form as many diverse teams as possible. We show that, for each efficient matching, there exists a vector of transfers satisfying an equal treatment of equals property that stabilize the matching (Proposition 2).

Second, we consider the case in which teammates must divide team output equally and non-cooperatively choose the quantity of information they produce. Our first contribution is to characterize the Nash equilibrium correspondence of the signal-acquisition game played within teams in order to determine each team's payoff frontier (Proposition 3). We identify a cutoff value on the (state-conditional) pairwise correlation between workers' signals that orders within-team Nash equilibria in terms of their symmetry. Intuitively, more positively correlated signals contain more redundant information. Thus, the marginal value of producing a signal when one's teammate has already produced one is decreasing in correlation. It follows that, when signals are positively correlated, i.e., the team is homogeneous, there is a unique asymmetric equilibrium up to worker identity. In it, one worker produces all of the team's information, while the other free-rides off her production. Conversely, when signals are negatively correlated, i.e., the team is diverse, there is a unique symmetric equilibrium in which effort is matched (Proposition 3). Because of the close relationship between efficient and equilibrium signal-acquisition profiles, it turns out that every efficient allocation is stable, a possible reason why a manager might choose to delegate sorting to her workers in the first place (Proposition 4).

Nevertheless, under the assumption that workers share the returns of their information equally, inefficient matchings may also be stable. We formally define the two sources of inefficiency — stratification inefficiency and asymmetric effort inefficiency. Our main result, Proposition 5, identifies necessary and sufficient conditions under which each arises. Our characterization has two parts. First, we show that whether stratification inefficiency or asymmetric inefficiency arises depends on the degree to which diverse teams can exploit informational complementarities. Specifically, there is a cutoff correlation below which a worker would rather form a diverse team than free-ride in a homogeneous team, and above which the opposite holds. Below this cutoff, all inefficient and stable allocations are characterized by stratification inefficiency, while, above it, all inefficient and stable allocations are characterized by asymmetric effort inefficiency. Second, we provide necessary and sufficient conditions on the correlation structure among workers that result in each inefficiency. Stratification inefficiency arises precisely when two diverse teams can be formed, but there exists a diverse team whose formation would cause excluded workers to form a homogeneous team. Outside of a peculiar case, asymmetric effort inefficiency arises precisely when it is feasible to form two homogeneous teams and select equilibria within these teams such that those producing all signals in the team cannot form a diverse team.

1.4 Related Literature

Team Theory. Chade and Eeckhout (2018) study the optimal assignment of workers to teams in the same (canonical) Gaussian environment that we consider, but with two important differences: (i) each worker produces exactly one signal within a team and (ii) utility is transferable. In our environment, in contrast to (i), workers can acquire any number of signals and, in contrast to (ii), utility is imperfectly transferable. This first difference means that the utility of workers in a team are affected not only by their pairwise correlation, but also the number of signals each worker (endogenously) acquires (see (1), which subsumes the formula for the value of a team in Chade and Eeckhout (2018) when each worker acquires a single signal). The second difference allows us to study the impact of moral hazard on sorting, a "relevant open problem with several economic applications" (Chade and Eeckhout, 2018). Our analysis, consequently, focuses on the *efficiency* of equilibrium teams as opposed to their assortativity, which itself is difficult to define in our setting.

An additional difference between our setup and that of Chade and Eeckhout (2018) is that they assume that signals between workers possess a *common* correlation parameter, but differ in variance, whereas we assume the opposite. We make this assumption to capture research settings in which workers are identical in their level of "expertise", but may come from different backgrounds. Our work, therefore, contributes to the literature on diversity in teams, i.e., Prat (2002), Hong and Page (2001), and Hong and Page (2004).[4] In particular, asymmetric effort inefficient allocations are characterized by excessive homogeneity, i.e., high correlation, within teams. Our results thus illustrate a new channel through which moral hazard can cause homogeneous teams to form even diverse teams are efficient.

Sorting and Bilateral Moral Hazard. Legros and Newman (2007) consider general two-sided matching environments in which, for each matched pair, there is an exogenously specified utility possibility frontier.[5] Our paper joins a small literature that considers

[4] Prat (2002) finds conditions under which a team should be composed of homogeneous information structures when these information structures are priced according to market forces. Hong and Page (2001) and Hong and Page (2004) consider the performance of heterogeneous non-Bayesian problem solvers. In contrast, we consider the *endogenous* formation of teams by Bayesian workers within a firm with a fixed information structure.

[5] A well-known application of this framework is to risk-sharing within households. Legros and Newman (2007) and Chiappori and Reny (2016) show that if couples share risk efficiently, then all stable matchings are negative assortative. Gierlinger and Laczó (2018) show that if the assumption of perfect risk-sharing is relaxed, then positive assortative matching can occur. Schulhofer-Wohl (2006) finds necessary and sufficient conditions for preferences under which risk-sharing problems admit a transferable utility represen-

matching settings in which the utility possibility frontier of each matched pair is affected by the presence of bilateral moral hazard.[6] Kaya and Vereshchagina (2015) study one-sided matching between partners who, after matching, play a repeated game with imperfect monitoring (due to moral hazard) and transfers. While moral hazard limits the achievable joint surplus attainable by a matched pair, transfers ensure that the Pareto-frontier is linear, i.e., payoffs are transferable. Hence, stable matchings exist and (constrained) efficiency is ensured by standard arguments, in contrast to our setting.[7]

Vereshchagina (2019) studies two-sided matching between financially-constrained entrepreneurs in the presence of bilateral moral hazard and incomplete contracts; entrepreneurs can only sign contracts under which the realized revenue is split between the partners according to an equity-sharing rule.[8] Non-transferability of output gives rise to inefficient positive sorting through the following channel: wealthy entrepreneurs, whom contribute more resources to joint production, are willing to form partnerships with poor entrepreneurs only if they receive a high equity share. But, joint surplus maximizing equity shares may be constant across all partnerships. Hence, wealthy entrepreneurs prefer to match even if the overall benefit of re-matching with poor entrepreneurs is large. The logic behind inefficiency thus resembles that of stratification inefficiency. We note, however, that there is no analog to asymmetric effort inefficiency in her model.

Finally, Kräkel (2017) considers a very different channel through which moral hazard leads to inefficient endogenous sorting. He studies an environment in which a firm posts an initial contract that determines both wages and a sorting protocol (workers either endogenously sort into teams or are randomly assigned to teams). The firm then receives interim information about the efficiency of the matches formed and can re-negotiate the initial contract. Under endogenous sorting, workers may form inefficient teams in order to force the firm to re-negotiate the initial contract.

Correlation and Information Acquisition. More broadly, our analysis of the information

tation.

[6]Wright (2004), Serfes (2005), Serfes (2007), and Sperisen and Wiseman (2016) study the assortativity of stable matchings in the presence of one-sided moral hazard, i.e., principals matching agents. For more recent contributions to this literature, see Section 5.2 of Chade and Swinkels (2020) and Chade and Eeckhout (2022).

[7]Kaya and Vereshchagina (2014) study a special case of their model in which workers form partnerships that may involve "money burning" to provide incentives. They then ask whether workers would prefer to work for an entrepreneur, i.e., hire a budget-breaker, as in Franco, Mitchell and Vereshchagina (2011) to avoid this problem. Chakraborty and Citanna (2005) consider a model similar to that of Kaya and Vereshchagina (2015) in which partners play asymmetric roles.

[8]Two-sidedness again ensures that a stable matching exists, in the sense of Legros and Newman (2007), unlike in our setting.

acquisition game played within teams is related to recent work defining notions of complementary and substitutable information. In the environment we consider, lower correlation implies higher complementarity in terms of the value of information. Börgers, Hernando-Veciana and Krähmer (2013) define signals as complements or substitutes in terms of their value across *all* decision problems, therefore requiring stronger conditions. Liang and Mu (2020) adapt the definition of Börgers, Hernando-Veciana and Krähmer (2013) to a multivariate Gaussian environment and use it characterize the learning outcomes of a sequence of myopic players.

2 Model

There are four workers, indexed by the set $\mathcal{N} := \{1,2,3,4\}$, who cooperatively form teams. Each team completes a different project and exactly two workers are required to complete each project. Therefore, any feasible assignment of workers is a (matching) function $\mu : \mathcal{N} \to \mathcal{N}$ with the property that the teammate of worker i's teammate, j, is i (that is, if $j = \mu(i)$, then $\mu(j) = i$) and that no worker is unassigned (that is, $\mu(i) \neq i$ for all $i \in \mathcal{N}$). Let $\mathcal{M}$ denote the set of all such functions.

2.1 Discrete Signal-Acquisition Game

Each project involves guessing the value of a state θ, which has a standard Gaussian distribution. We first define a game, parameterized by K, in which each worker acquires unbiased, conditionally independent Gaussian signals with variance $\frac{\sigma^2}{K}$. A worker incurs an effort cost of $\frac{c}{2}$ to produce a signal, where $0 < c < \min\{\frac{1}{8}\sigma^{-2}, \sigma^2\}$.[9] The number of signals worker i produces, n_i, belongs to a grid $\{0,1,2,...,KM\}$, where $M > \sqrt{\sigma^2}/\sqrt{c}$, and is unobservable to the firm. The condition on the cost of effort ensures that at least one worker has an incentive to acquire a strictly positive number of signals and that acquiring negatively correlated signals is sufficiently valuable. It also ensures that a stable matching exists under transferable utility. The choice of M ensures that no worker has an incentive to acquire more than M signals in any team.

The signals of workers in the same team are correlated. For simplicity, we assume that, for any distinct workers i and j, $\rho_{ij} \in \{\rho_\ell, \rho_h\}$ is the state-conditional correlation

[9]The assumption that marginal costs are constant is not crucial for our main results. Kambhampati and Segura-Rodriguez (2022) assumes that the cost function satisfies increasing marginal costs and obtains a qualitatively similar within-team equilibrium characterization.

coefficient between worker i's and worker j's signals. In addition, we assume that (i) $-1 < \rho_\ell < 0 < \rho_h < 1$ and (ii) for each $\rho \in \{\rho_\ell, \rho_h\}$ there exist workers i and j with $\rho_{ij} = \rho$. We call a team (i, j) **diverse** if $\rho_{ij} = \rho_\ell$, as workers in such a team produce negatively correlated signals, and **homogeneous** if $\rho_{ij} = \rho_h$, as workers in such a team produce positively correlated signals. While the use of negative correlation as a measure of task-related diversity has precedence in the literature (see, e.g., Kvaløy and Olsen (2019)), it is important to emphasize that the terms "diverse" and "homogeneous" should be interpreted in relative, rather than absolute, terms. A team with a positive correlation coefficient close to zero might be considered a "diverse" team relative to teams outside of the firm.

The correlation structure captures the economics of a situation in which matched signals are affected by complementarities, while unilateral signals are not. In particular, if $n_i \geq n_j > 0$, then workers i and j produce n_j conditionally correlated signals and worker i produces $n_i - n_j$ signals, each of which is conditionally uncorrelated with all other signals.[10] After observing the signal realizations of every team member, team (i, j) takes an action $a^* \in \mathbb{R}$ to minimize the expected value of a quadratic loss function. Formally,

$$a^* \in \arg\max_{a \in \mathbb{R}} E_\theta \left[1 - (a - \theta)^2 \mid x^S \right],$$

where x^S denotes the concatenation of signals observed in the team. Each worker obtains an equal share of the project's profit, $1 - (a - \theta)^2$. We view this as a descriptively plausible assumption, rather than a result of optimal contracting.

A reader may now find an economic application useful. Consider a manufacturing firm that wants to estimate the cost of production of its next product. Suppose the task requires two workers and there are two types of workers, product engineers and data scientists. Each worker in a team decides how many prototypes inspect (prototypes must be inspected in a fixed order) and obtains one cost estimate per prototype. The final cost estimate of the team is a weighted average of all estimates obtained by the two workers. Engineers and data scientists differ in their methodological expertise: an engineer might obtain a cost estimate from a prototype using on-the-job experience, while a data scientist might rely on market research. Hence, two engineers (or two data scientists) might acquire positively correlated cost estimates from a given prototype, while a diverse team

[10] Diminishing marginal returns and complementarity are separate forces shaping workers' information acquisition strategies in our model. While acquiring more signals reduces the marginal productivity of acquiring one's own signals, correlation between signals across workers captures the degree of complementarity of information. These are plausible forces in information-producing teams that, nevertheless, are not easy to analyze separately in reduced-form production models. Our specification allows us to analyze how signal acquisition is shaped by complementarities, holding fixed individual marginal returns.

composed of an engineer and a data scientist might obtain negatively correlated estimates.

2.2 Continuous Limit Game

In a K-discrete game, worker i's payoff in team (i,j) is[11]

$$u_i(n_i, n_j; \rho_{ij}) := \frac{1}{2}\left(1 - E_x\left[\min_{a\in\mathbb{R}} E_\theta\left[(a-\theta)^2 \mid x^S\right]\right]\right) - \frac{c}{2K} n_i.$$

By simplifying the posterior variance, $E_x\left[\min_{a\in\mathbb{R}} E_\theta\left[(a-\theta)^2 \mid x^S\right]\right]$, Appendix A.1 establishes that worker i's payoff function is equal to

$$u_i(n_i, n_j; \rho_{ij}) := \frac{1}{2}\left(1 - \frac{\sigma^2}{K}\left(\underline{n}_{ij}\left(\frac{1-\rho_{ij}}{1+\rho_{ij}}\right) + \bar{n}_{ij} + \frac{\sigma^2}{K}\right)^{-1}\right) - \frac{c}{2K} n_i, \tag{1}$$

where $\underline{n}_{ij} = \min\{n_i, n_j\}$ and $\bar{n}_{ij} = \max\{n_i, n_j\}$. Notice that if n_i is interpreted as producing $\frac{n_i}{K}$ signals, then we can re-write (1) as

$$v_i(n_i, n_j; \rho_{ij}) := \frac{1}{2}\left(1 - \sigma^2\left(\underline{n}_{ij}\left(\frac{1-\rho_{ij}}{1+\rho_{ij}}\right) + \bar{n}_{ij} + \sigma^2\right)^{-1}\right) - \frac{1}{2} c n_i. \tag{2}$$

Put differently, each K-discrete game is strategically equivalent to one in which worker i's strategy space is $\{0, \frac{1}{K}, \ldots, M\}$ and her payoff function is defined by Equation 2. To ease notation, we denote $n_i(j)$ and $n_j(i)$ by n_i and n_j and drop the dependence of v_i in Equation 2 on ρ_{ij} whenever there is no confusion that j is i's teammate.

We now define a continuous limit game, obtained as $K \to \infty$. Specifically, we define a normal-form game, called the **Production Subgame**, in which worker i's strategy space is the interval $[0, M]$ and her payoff function is defined by Equation 2. In Appendix A.2, we prove that, as $K \to \infty$, the strategy space for worker i, $\{0, \frac{1}{K}, \ldots, M\}$, converges to the interval $[0, M]$ and the set of Nash equilibria converges to the set of Nash equilibria in the continuous game. Hence, the Production Subgame is the limit of the K-discrete games and strategic behavior is appropriately interpreted as an approximation of strategic behavior in nearby, fractional signal-acquisition games. Our analysis focuses on the limit game due to its superior tractability.[12]

[11] We remark that the payoff functions are consistent with any setting in which the employer retains a fixed share of the project's profit $\alpha \in (0,1)$, with the residual share, $1-\alpha$, divided equally among workers in the team.

[12] In a previous working paper Kambhampati, Segura-Rodriguez and Shao (2021), we analyze the properties of K-discrete games and characterize their (qualitatively similar) equilibria.

2.3 Solution Concept

A signal-acquisition strategy for worker i is a function mapping teammate identity to a non-negative real number of signals, $n_i : \mathcal{N} \backslash \{i\} \to [0, M]$. Denote the profile of signals chosen within team (i,j) by $n(i,j) := (n_i(j), n_j(i))$. In each team (i,j), we require that the strategy profile $n^*(i,j)$ is a Nash equilibrium of the corresponding Production Subgame. For the two-stage game, we use stability as our solution concept, the standard solution concept in the literature on matching with imperfectly transferable utility.[13] In these settings, an **allocation** is a pair (μ, N^*), where $\mu \in \mathcal{M}$ is a matching and $N^* = \{n^*(i,j)\}_{i,j=\mu(i)}$ is a collection of within-team Nash equilibria. An allocation is **stable** if no pair can match and play a Nash equilibrium that makes both strictly better off than under the initial allocation.

3 Perfectly Transferable Utility Allocations

We first consider the sorting of workers into teams when teammates can coordinate on their signal-acquisition profile and freely divide team surplus.

3.1 Marginal Value of Information

To develop intuition about efficient signal acquisition, we first define and analyze the **marginal value of information** generated by worker i given a signal profile $n(i,j)$ in a team with correlation coefficient ρ. Upon observing such a vector of signals, the team's posterior variance about the state — its **ex-post variance** — is

$$\sigma^2 \left(\underline{n}_{ij} \left(\frac{1-\rho}{1+\rho} \right) + \bar{n}_{ij} + \sigma^2 \right)^{-1}.$$

So, the marginal value of information produced by worker i, i.e., the marginal reduction in ex-post variance, is

$$MV(n_i; n_j, \rho) := \sigma^2 \left(\frac{\frac{\partial}{\partial n_i} \left(\underline{n}_{ij} \left(\frac{1-\rho}{1+\rho} \right) + \bar{n}_{ij} \right)}{\left(\underline{n}_{ij} \left(\frac{1-\rho}{1+\rho} \right) + \bar{n}_{ij} + \sigma^2 \right)^2} \right) > 0,$$

where the partial derivative is taken to be the right derivative if $n_i = n_j$. If $n_i \geq n_j$, we call worker i a **high producer**. In this case, additional information produced by worker

[13] See Legros and Newman (2007) for a general definition in two-sided environments and Kaya and Vereshchagina (2015) for a definition in a one-sided environment.

i is novel; it is uncorrelated with existing information. Consequently, the marginal value of information is affected by the correlation coefficient between the two workers only through the value of existing information:

$$MV(n_i; n_j, \rho) = \sigma^2 \left(\underline{n}_{ij} \left(\frac{1-\rho}{1+\rho} \right) + \bar{n}_{ij} + \sigma^2 \right)^{-2}.$$

Conversely, if $n_i > n_j$, we call worker j a **low producer**. Consequently, the marginal value of information for j is affected by the correlation coefficient between the two workers both through the value of existing information and the value of matching worker i's information:

$$MV(n_j; n_i, \rho) = \sigma^2 \left(\underline{n}_{ij} \left(\frac{1-\rho}{1+\rho} \right) + \bar{n}_{ij} + \sigma^2 \right)^{-2} \left(\frac{1-\rho}{1+\rho} \right).$$

Notice that, given a signal profile $n(i,j)$, the marginal value of information produced by a high producer is higher than the marginal value of information produced by a low producer in a homogeneous team with $\rho = \rho_h > 0$. The opposite relationship holds for a diverse team with $\rho = \rho_\ell < 0$.

3.2 Efficient Signal Acquisition

In a team with pairwise correlation ρ, an **efficient profile of signals** is one that maximizes the total of surplus of the team. That is, it solves

$$V(\rho) := \max_{n_i, n_j \in [0,M]} \left(1 - \sigma^2 \left(\underline{n}_{ij} \left(\frac{1-\rho}{1+\rho} \right) + \bar{n}_{ij} + \sigma^2 \right)^{-1} \right) - \frac{1}{2} c n_i - \frac{1}{2} c n_j.$$

Because the marginal cost of acquiring a signal is constant across workers, it can only be efficient for the worker with the highest marginal value of information to acquire additional information. Hence, for a homogeneous team with $\rho = \rho_h > 0$, it is efficient for only one of the two workers to acquire a strictly positive number of signals. On the other hand, for a diverse team with $\rho = \rho_\ell < 0$, efficient signal profiles are symmetric. The following Proposition identifies efficient signal profiles in each type of team and characterizes total surplus.

Proposition 1 (Efficient Signal Acquisition)**.** *Fix a team (i, j) with correlation coefficient $\rho \in (-1, 1)$. Then, the following properties hold:*

1. *If $\rho > 0$, i.e., team (i, j) is homogeneous, then there are two efficient signal profiles. In*

one,

$$n_i = 0 \quad \textit{and} \quad n_j = \sqrt{\frac{2\sigma^2}{c}} - \sigma^2.$$

In the other, $n_i = \sqrt{\frac{2\sigma^2}{c}} - \sigma^2$ *and* $n_j = 0$*. Efficient total surplus is*

$$V(\rho) = 1 + \frac{c\sigma^2}{2} - \sqrt{2c\sigma^2}.$$

2. *If* $\rho = 0$*, then a profile* $n(i,j)$ *is efficient if and only if*

$$n_i + n_j = \sqrt{\frac{2\sigma^2}{c}} - \sigma^2.$$

   Efficient total surplus is

$$V(\rho) = 1 + \frac{c\sigma^2}{2} - \sqrt{2c\sigma^2}.$$

3. *If* $\rho < 0$*, i.e., team* (i,j) *is diverse, then the unique efficient profile* $n(i,j)$ *has*

$$n_i = n_j = \left(\frac{1+\rho}{2}\right)\left(\sqrt{\frac{1}{(1+\rho)}}\sqrt{\frac{2\sigma^2}{c}} - \sigma^2\right).$$

   Efficient total surplus is

$$V(\rho) = 1 + \frac{c\sigma^2}{2}(1+\rho) - \sqrt{2c\sigma^2(1+\rho)}.$$

Proof. See Appendix A.3. □

3.3 Efficient Matching

We now establish an equivalence between efficient matchings, maximally diverse matchings, and stable matchings. Moreover, we characterize the division of surplus within each team that supports each efficient matching as a stable matching.

A matching $\mu \in \mathcal{M}$ is **TU-efficient** if it solves

$$\max_{\mu \in \mathcal{M}} \quad V(\rho_{i\mu(i)}) + V(\rho_{j\mu(j)}) \quad \text{for } j \neq i \text{ and } j \neq \mu(i).$$

It is **maximally diverse** if it forms as many diverse teams as possible, i.e., there does not exist another matching $\hat{\mu} \in \mathcal{M}$ for which

$$|\{i \in \mathcal{N} : \rho_{i\hat{\mu}(i)} = \rho_\ell\}| > |\{i \in \mathcal{N} : \rho_{i\mu(i)} = \rho_\ell\}|.$$

Finally, it is **TU-stable** if there exists a positive vector of transfers $v \in \mathbb{R}^4_+$ such that

$$v_i + v_{\mu(i)} \leq V(\rho_{i\mu(i)}) \quad \text{for all } i \tag{F}$$

and

$$v_i + v_j \geq V(\rho_{ij}) \quad \text{for all } i \text{ and } j \neq i. \tag{S}$$

The first condition ensures that the sum of within-team transfers does not exceed the matching surplus. The second condition ensures that no two teammates can form a team and divide their surplus in a way that yields each a strictly higher payoff.

To characterize stabilizing transfers, it will be useful to define $G_\ell = (V, E)$ to be the (simple) graph with vertices equal to the set of workers, $V := \mathcal{N}$, and edges linking workers whom compose diverse teams,

$$E := \{(i,j) | \; i,j \in V, \;\; i < j, \text{ and } \rho_{ij} = \rho_\ell\}.$$

By assumption, G_ℓ satisfies $1 \leq |E| \leq 5$. Hence, there are a total of 62 possible graphs. Nevertheless, there are only nine graphs up to isomorphism. Formally, $G^1_\ell = (V^1, E^1)$ is **isomorphic** to $G^2_\ell = (V^2, E^2)$ if there exists a one-to-one and onto map $\psi : V^1 \to V^2$ such that $(i,j) \in E^1$ if and only if $(\psi(i), \psi(j)) \in E^2$. It can be easily shown that every feasible graph, G_ℓ, is isomorphic to exactly one of the nine graphs depicted in Figure 1. Finally, we say that two workers i and j are of the same **type** if $\rho_{ik} = \rho_{jk}$ for any $k \neq i,j$. A vector of transfers $v \in \mathbb{R}^4_+$ then satisfies **equal treatment of equals** if $v_i = v_j$ whenever i and j are of the same type.

We now state our result.

Proposition 2 (TU-Efficient Matching)**.**

1. *A matching is TU-efficient if and only if it is maximally diverse if and only if it is TU-stable.*

2. *Any TU-efficient matching is supported as a TU-stable matching by a vector of transfers that respects equal treatment of equals.*

Proof. See Appendix A.4. □

Because the difference in surplus between a diverse team and a homogeneous team, $V(\rho_\ell) - V(\rho_h)$, is strictly positive for any $\rho_\ell \in (0,1)$, it is immediate that efficiency calls for maximal diversity. However, the equivalence between TU-efficient and TU-stable matchings is not immediate because the environment is one-sided. For instance, if G_ℓ

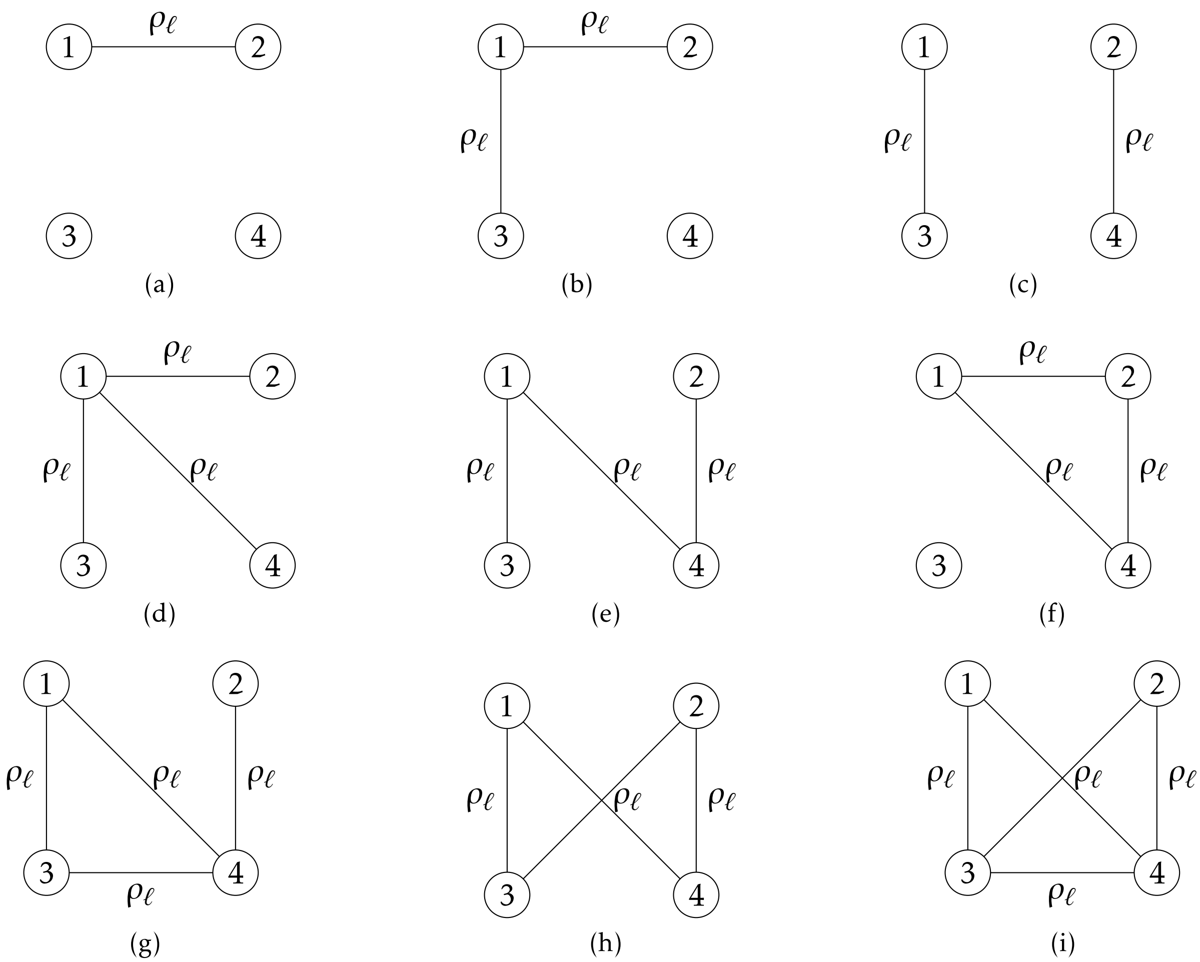


Figure 1: Isomorphism classes of correlation graphs.

is isomorphic to Figure 1f, ρ_ℓ is sufficiently small, and $c > \frac{1}{8}\sigma^{-2}$ (a case ruled out by assumption), then three TU-efficient matchings exist, but there does not exist a TU-stable matching (the problem is akin to a transferable-utility version of the roommate problem of Gale and Shapley (1962)). So, it is directly proven that every TU-efficient matching can be stabilized by an appropriately defined transfer vector and that if a matching is TU-inefficient, then no such transfers exist.

We discuss here the stabilizing transfers. If G_ℓ is isomorphic to Figure 1c, 1e, 1g, 1h, or 1i, then any maximally diverse (and hence, efficient) matching forms two diverse teams. When each worker receives $\frac{1}{2}V(\rho_\ell)$ — a transfer vector that trivially respects equal treatment of equals — then no two workers can form a profitable deviating team and each obtain a strictly higher utility (no homogeneous team can generate more than $V(\rho_\ell)$ total surplus). If G_ℓ is isomorphic to Figure 1a, then the unique maximally diverse matching has one diverse team and one homogeneous team. Under the unique transfer vector exhausting all surplus and respecting equal treatment of equals, two workers receive $\frac{1}{2}V(\rho_h)$ and two workers receive $\frac{1}{2}V(\rho_\ell)$. Given the graph structure, however, any deviating team must be homogeneous. So, the sum of surpluses exceeds the value of the team: $\frac{1}{2}V(\rho_h) + \frac{1}{2}V(\rho_\ell) > V(\rho_h)$ by $V(\rho_h) < V(\rho_\ell)$. It follows that the TU-efficient matching is TU-stable.

In the remaining graphs, TU-stability is maintained by providing workers of the same type the same utility, even when their assigned teams generate asymmetric levels of surplus. For instance, if G_ℓ is as depicted in Figure 1f, then the transfers $v_1 = v_3 = v_4 = \frac{1}{2}V(\rho_\ell)$ and $v_2 = V(\rho_h) - \frac{1}{2}V(\rho_\ell)$ stabilize any TU-efficient matching. Crucially, any worker forced to work in a homogeneous team that is capable of forming a diverse team is paid a compensating differential to account for the decline in their match value (they receive more than half of the surplus generated in the homogeneous team). To see that the constructed transfers are positive, it is useful to note that $c < \frac{1}{8}\sigma^{-2}$ implies that total surplus generated by a diverse team, $V(\rho_\ell)$, is less than twice that of a homogeneous team, $V(\rho_h)$.

4 Imperfectly Transferable Utility Allocations

We now consider the sorting of workers into teams when teammates must divide team output equally and non-cooperatively choose the quantity of information they produce.

4.1 Production Subgame Analysis

We first characterize Nash equilibria played within a team (i, j) with arbitrary correlation $\rho \in (-1, 1)$. In what follows, let $S(\rho)$ denote the total surplus generated by a team with correlation ρ in a surplus maximizing equilibrium.

Proposition 3. *Fix a team (i, j) with correlation coefficient $\rho \in (-1, 1)$. Then, the following properties hold:*

1. *If $\rho > 0$, i.e., team (i, j) is homogeneous, then there are two Nash equilibria. In one,*

$$n_i = 0 \quad \text{and} \quad n_j = \sqrt{\frac{\sigma^2}{c}} - \sigma^2.$$

   In the other, $n_i = \sqrt{\frac{\sigma^2}{c}} - \sigma^2$ and $n_j = 0$. The following payoff vectors are therefore feasible:

$$\gamma_1 := \left(\frac{1}{2}(1 + c\sigma^2 - 2\sqrt{c\sigma^2}), \frac{1}{2}(1 - \sqrt{c\sigma^2})\right) \quad \text{and} \quad \gamma_2 := \left(\frac{1}{2}(1 - \sqrt{c\sigma^2}), \frac{1}{2}(1 + c\sigma^2 - 2\sqrt{c\sigma^2})\right).$$

   The difference between efficient and equilibrium total surplus is

$$V(\rho) - S(\rho) = \left(\frac{3}{2} - \sqrt{2}\right)\sqrt{c\sigma^2}.$$

2. *If $\rho = 0$, then a profile $n(i, j)$ is a Nash equilibrium if and only if*

$$n_i + n_j = \sqrt{\frac{\sigma^2}{c}} - \sigma^2.$$

   The following payoff vectors are therefore feasible:

$$\left\{\alpha \cdot \gamma_1 + (1 - \alpha) \cdot \gamma_2, \quad \alpha \in [0, 1]\right\}.$$

   The difference between efficient and equilibrium total surplus is

$$V(\rho) - S(\rho) = \left(\frac{3}{2} - \sqrt{2}\right)\sqrt{c\sigma^2} > 0.$$

3. *If $\rho < 0$, i.e., team (i, j) is diverse, then the unique Nash equilibrium has*

$$n_i = n_j = \left(\frac{1 + \rho}{2}\right)\left(\sqrt{\frac{1 - \rho}{1 + \rho}}\sqrt{\frac{\sigma^2}{c}} - \sigma^2\right).$$

   The unique feasible payoff vector has each worker achieve a payoff of

$$\frac{1}{2}\left(1 + \frac{c\sigma^2}{2}(1 + \rho) - \left(\frac{3 - \rho}{2\sqrt{1 - \rho}}\right)\sqrt{c\sigma^2(1 + \rho)}\right).$$

The difference between efficient and equilibrium total surplus is

$$V(\rho) - S(\rho) = \left(\frac{3-\rho}{2\sqrt{1-\rho}} - \sqrt{2} \right) \sqrt{c\sigma^2(1+\rho)} > 0.$$

Because this difference is maximized as $\rho \uparrow 0$, the total surplus reduction is smaller in a diverse team than in a homogeneous team.

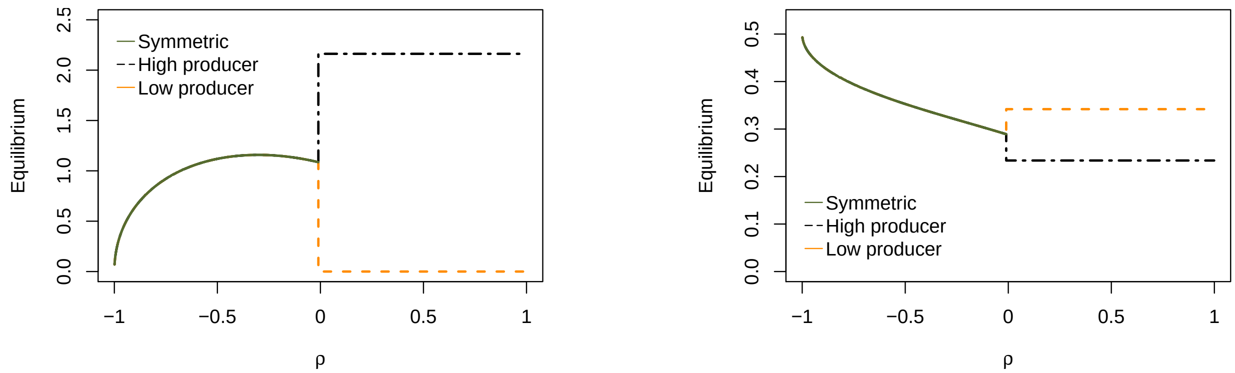
Proof. See Appendix A.5. □

From Proposition 3, we see that, except when $\rho = 0$, there exists a unique Nash equilibrium (up to worker identity). Interestingly, for negative correlation coefficients, this equilibrium is symmetric, while for positive coefficients it is asymmetric.[14] The intuition is simple: When the correlation coefficient is negative, complementarities make it more valuable for a low producer to increase his effort up to the level of a high producer than for a high producer to increase his amount of effort. The opposite intuition holds for positive correlation coefficients: Independently of how many signals each worker produces, the high producer's marginal value of information is always higher than the low producer's marginal value of information. Finally, observe that budget-balanced division of output comes into conflict with surplus maximization when effort is non-contractible (Holmström (1982)). We observe, however, that the reduction in total surplus from the equal-sharing rule ends up being *larger* in homogeneous teams than in diverse teams. Hence, a priori, it is unclear that the equal-sharing rule may lead to fewer diverse teams.

Figure 2a illustrates our results. The team's correlation parameter, ρ, is on the x-axis, while Nash equilibrium strategies are on the y-axis. The solid, green line indicates the strategy of each worker in a symmetric Nash equilibrium; the symmetrically-spaced, dashed, orange line indicates the strategy of a low producer in an asymmetric Nash equilibrium; and the asymmetrically-spaced, dashed, black line indicates the strategy of a high producer in an asymmetric Nash equilibrium.

Figure 2b presents corresponding Nash equilibrium payoffs. From Proposition 3, we see that, for any value of $\rho \in (-1, 1)$, a team is associated with a unique *total* surplus value. However, due to nontransferable effort costs, workers cannot freely divide this surplus. When choosing a teammate, each worker thus takes into account not only the total surplus the team generates, but how much of it she can achieve. The following corollary

[14] We remark that while the Nash equilibrium in homogeneous teams involves one worker acquiring zero signals, this property need not hold in related, discrete signal-acquisition games (see Kambhampati, Segura-Rodriguez and Shao (2021)). We interpret the lower bound on effort of zero as a "minimum" amount of effort that a worker can exert.



(a) Equilibrium Correspondence.

(b) Payoff Correspondence.

Figure 2: Equilibrium Correspondence with $\sigma^2 = 1$ and $c(n) = 0.1n$.

states a number of useful properties of feasible payoff vectors, necessarily satisfied in Figure 2a, that will be useful when we analyze equilibrium sorting.

Corollary 1.

1. *In any diverse team, the unique feasible payoff vector is strictly decreasing (in both components) in the team's correlation coefficient.*

2. *In any homogeneous team, the set of feasible payoff vectors is constant in the team's correlation and, in any Nash equilibrium in the team, the low producer obtains a strictly higher utility than the high producer.*

3. *A high producer in a homogeneous team always obtains a strictly lower utility than she would in a diverse team.*

4. *There exists a unique value $\rho^* \in (-1, 0)$ such that, in any diverse team, each worker would obtain a higher payoff as a low producer in a homogeneous team if and only if $\rho_\ell \geq \rho^*$.*

Proof. See Appendix A.6. □

The proof follows from simple algebraic manipulation. Property 4 of Corollary 1 will be particularly important when characterizing equilibrium sorting patterns: A worker may sometimes prefer to join a homogeneous team, which produces less valuable information than a diverse team, if she can save on effort costs.

4.2 Efficient Allocations are Stable

We now characterize which allocations are (constrained) efficient and show that these allocations are stable. Informally, an allocation is efficient if there does not exist another matching and collection of equilibria that strictly increases utilitarian welfare. A formal definition follows below.

Definition 1 (Efficient Allocations)**.** *An allocation (μ, N^*) is **efficient** if there does not exist a matching $\hat{\mu} \in \mathcal{M}$ and a collection of Nash equilibria $\hat{N} = \{\hat{n}(i,j)\}_{i,j=\mu(i)}$ for which*

$$\sum_{\ell \in \mathcal{N}} v_\ell(n^*(\ell, \mu(\ell))) < \sum_{\ell \in \mathcal{N}} v_\ell(\hat{n}(\ell, \hat{\mu}(\ell))).$$

*It is **inefficient** otherwise.*

We observe that the set of efficient allocations is equivalent to the set of maximally diverse allocations. Moreover, there always exists an efficient, i.e., maximally diverse, stable allocation. The result is intuitive given the observation that the within-team free-riding problem harms homogeneous teams more than diverse teams and that efficient matchings are always sustained as TU-stable matchings under transfers respecting equal treatment of equals.

Proposition 4 (Characterization of Efficient Allocations)**.**

1. *An allocation (μ, N^*) is efficient if and only if μ is maximally diverse.*

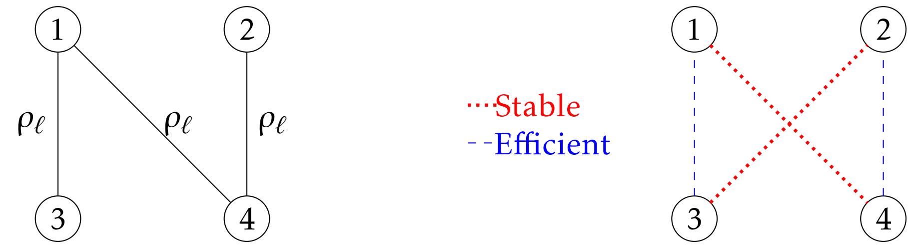
2. *Every efficient allocation is stable.*

Proof. See Appendix A.7. □

That maximally diverse teams remain stable even under imperfectly transferable utility provides an explanation for why a manager might choose to delegate sorting to her workers. However, as we next show, restrictions on transfers give rise to stable allocations that are *inefficient*.

4.3 Stable and Inefficient Allocations

Our analysis of the Production Subgame yields two important insights. First, fixing a strategy profile within teams, reducing correlation increases the value of information the team generates. Hence, there is a tendency for workers with a low pairwise correlation to match, ignoring effort costs. Second, increasing correlation decreases the symmetry



(a) Correlation graph G_ℓ. (b) Sorting when $\rho_\ell < \rho^*$.

Figure 3: Stratification Inefficiency.

of equilibria; as signals become more substitutable, the marginal value of matching a high producer's signal decreases. Hence, the existence of nontransferable effort costs may tempt workers with a low pairwise correlation to join less productive teams if they can free-ride on their teammate's information. We now show how these two within-team properties can lead to inefficient sorting into teams.

4.3.1 Stable and Stratification Inefficient Allocations

We first exposit an inefficiency that arises when a diverse team forms, but causes excluded workers to form an inefficient, homogeneous team. In the graph depicted in Figure 3a, we argue that, if $\rho_\ell < \rho^*$ as defined in Corollary 1, then there is an inefficient stable allocation in which worker 1 and worker 4 form a diverse team, and worker 2 and worker 3 form a homogeneous team. To see why such an allocation is stable, observe that worker 1 and worker 4 play a symmetric Nash equilibrium, $n^*(1,4)$, while worker 2 and worker 3 play an asymmetric Nash equilibrium, $n^*(2,3)$, in which one worker does not exert any effort and the other exerts a strictly positive amount. By the assumption that $\rho_\ell < \rho^*$, however, worker 1 and worker 4 each obtains a weakly higher payoff together than they can achieve in any other Nash equilibrium in any other team. So, neither has a (strict) incentive to form a deviating team. Nevertheless, μ is *not* maximally diverse. In particular, if worker 1 forms a team with worker 3, and worker 2 forms a team with worker 4, then two diverse teams form instead of one. Thus, by Proposition 4, the original stable allocation could not have been efficient. See Figure 3b.

We now formalize the previous logic and define our first notion of inefficiency.

Definition 2 (Stratification Inefficiency)**.** *An allocation (μ, N^*) is* ***stratification inefficient*** *if μ is not a part of any efficient allocation and, in the unique Nash equilibrium in a diverse team, each worker obtains a strictly higher utility than any worker in any equilibrium in a* *homogeneous team.*

In a stratification inefficient allocation, two teammates are each as well off as in *any* other feasible team playing *any* other Nash equilibrium, e.g., worker 1 and worker 4. In addition, there exists another matching, e.g., $\hat{\mu}$, such that $\hat{\mu}(1) = 3$ and $\hat{\mu}(2) = 4$, and a collection of Nash equilibria that increases utilitarian welfare. Stratification inefficiency therefore arises when a diverse team forms, but does not internalize the effect it has on the productivity of the residual match.

Under what conditions does a stable and stratification inefficient allocation exist? The following result establishes that a stratification inefficient allocation is stable if and only if diverse teams are sufficiently productive and G_ℓ is isomorphic to one of three graphs in Figure 1.

Lemma 1. *There exists a stable and stratification inefficient allocation if and only if $\rho_\ell < \rho^*$ and G_ℓ is isomorphic to either Figure 1e, 1g, or 1i.*

Proof. See Appendix A.8. □

The economic intuition can be understood by inspecting how restrictions on transfers limit the formation of efficient "deviating" teams. In the correlation graph of Figure 3a (which is isomorphic to Figure 1e), worker 3 would be willing to form a team with worker 1 even if this meant that she would receive less than half of the equilibrium surplus generated by the team. However, given the lack of transfers, no profitable deviating team can form even when such a team is efficient. Similar issues arise if G_ℓ is isomorphic to Figure 1g or 1i. However, if G_ℓ is isomorphic to any other graph and a homogeneous team forms, then it is either part of an efficient matching or all workers belong to a homogeneous team. In the latter case, there is a profitable deviation — at least one diverse team exists by assumption and, by $\rho_\ell < \rho^*$, two workers in a homogeneous team strictly prefer to form a deviating diverse team.

4.3.2 Stable and Asymmetric Effort Inefficient Allocations

We now study a sorting inefficiency that arises due to asymmetric effort provision within teams. Consider the correlation graph depicted in Figure 4a and suppose that $\rho_\ell \geq \rho^*$, where ρ^* is defined in Corollary 1. We claim that there is an inefficient stable allocation in which only homogeneous teams form, even though any efficient matching forms only diverse teams. To see why such a matching can be stable, suppose that $\mu(1) = 2$ and

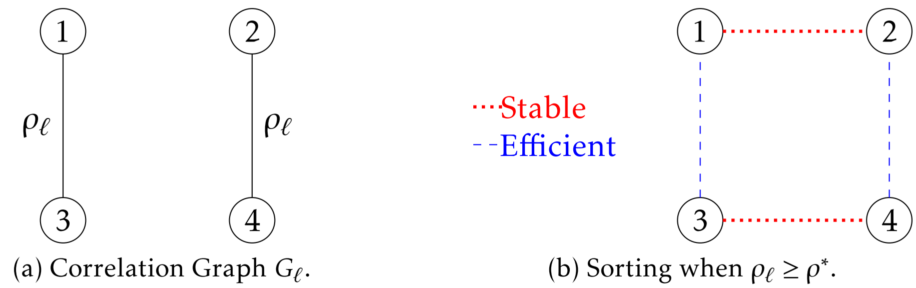


Figure 4: Asymmetric Effort Inefficiency.

$\mu(3) = 4$. Then, the unique equilibrium in the homogeneous teams $(1,2)$ and $(3,4)$ involves one worker producing a strictly positive number of signals and the other producing zero signals. Let $n_1^*(2) = 0 < n_2^*(1)$ and $n_4^*(3) = 0 < n_3^*(4)$, so that worker 1 and worker 4 produce zero signals. If $\rho_\ell \geq \rho^*$, worker 1 is unwilling to from a diverse team with worker 3 and worker 4 is unwilling to form a diverse team with worker 2 by property 4 of Corollary 1. Moreover, neither can do better in any other homogeneous team. Finally, worker 2 and worker 3 cannot form a team and strictly increase their payoffs; since $\rho_{23} = \rho_h$, one of them would, again, produce all signals in the team. It follows that the constructed allocation is stable.

We are now ready to define our second notion of inefficiency.

Definition 3 (Asymmetric Effort Inefficiency)**.** *An allocation (μ, N^*) is* ***asymmetric effort inefficient*** *if μ is not a part of any efficient allocation and, in the unique Nash equilibrium of a diverse team, each worker obtains a lower utility than some worker in any Nash equilibrium of a homogeneous team.*

To understand the definition, consider again the example. In it, worker 1 is willing to form a team with worker 2 because she obtains a strictly higher utility than she can in an efficient, diverse team with worker 3. The reason she obtains a higher utility is because she exerts less effort when matched with worker 2 than with worker 3.

Under what conditions does a stable and asymmetric effort inefficient allocation exist? We obtain the following result.

Lemma 2. *There exists a stable and asymmetric effort inefficient allocation if and only if $\rho_\ell \geq \rho^*$ and G_ℓ is isomorphic to Figure 1a, 1b, 1c, 1e, 1g, or 1i.*

Proof. See Appendix A.9. □

The economic intuition can again be understood by inspecting how restrictions on transfers limit the formation of efficient "deviating" teams. In the correlation graph of Figure 4a (which is isomorphic to Figure 1c), worker 2 would be willing to form a team with worker 4 — thereby disrupting the inefficient matching — even if this meant that she would receive less than half of the equilibrium surplus generated by the team. This could be done while yielding worker 4 a strictly larger utility than she receives in the stable allocation because the equilibrium surplus in a diverse team, $S(\rho_\ell)$, is larger than in a homogeneous team, $S(\rho_h)$. Similar issues arise in any graph G_ℓ that is *not* isomorphic to Figure 1d or 1h.

We now discuss why there is no asymmetric inefficient allocation if G_ℓ is isomorphic to Figure 1d, 1f, or 1h. In Figure 1d and 1f, every feasible matching is trivially efficient. A maximally diverse matching forms one diverse team and one homogeneous team. And, any feasible matching also forms one diverse and one homogeneous team. However, in Figure 1h, the logic is subtler. In the only inefficient matching, worker 1 matches with worker 2, and worker 3 matches with worker 4. In any Nash equilibrium in either team, one worker produces all signals and the other produces zero signals. However, in *any* such collection of equilibria, the two high producers can form a deviating diverse team and each be made strictly better off. Hence, the inefficient matching is unstable.

4.3.3 Complete Inefficiency Characterization

From the definitions of stratification inefficiency and asymmetric effort inefficiency, if $\rho_\ell < \rho^*$, then any inefficient allocation is stratification inefficient. Moreover, if $\rho_\ell \geq \rho^*$, then any inefficient allocation is asymmetric effort inefficient. Hence, an immediate consequence of Lemma 1 and Lemma 2 is a complete characterization of *all* inefficient allocations that may arise in the model. We state our portmanteau result below.

Proposition 5 (Complete Characterization of Inefficient and Stable Allocations)**.** *There exists an inefficient and stable allocation if and only if either*

1. *$\rho_\ell < \rho^*$ and G_ℓ is isomorphic to either Figure 1e, 1g, or 1i so that a stratification inefficient allocation exists; or,*

2. *$\rho_\ell \geq \rho^*$ and G_ℓ is isomorphic to Figure 1a, 1b, 1c, 1e, 1g, or 1i so that an asymmetric effort inefficient allocation exists.*

Notice that, for any value of ρ_ℓ, there exists a graph G_ℓ that leads to an inefficient and stable allocation. Whether the stable allocation is stratification inefficient or asymmetric effort inefficient depends on the productivity of the diverse teams in the organization. If these teams are sufficiently productive, then diverse teams may form at the expense of other workers. If these teams are sufficiently unproductive, then workers have opportunities to form homogeneous teams in order to minimize their effort.

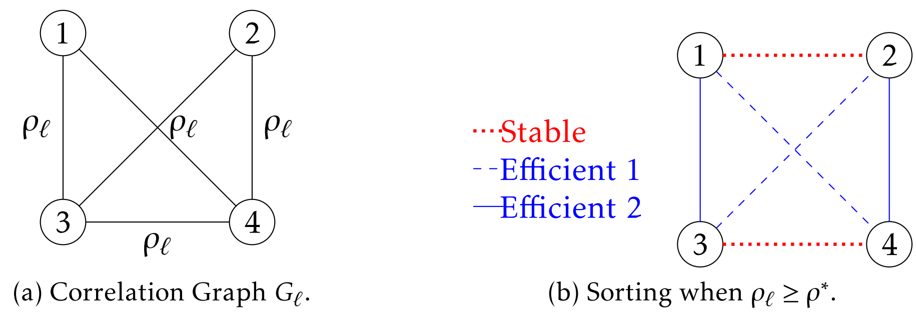
We conclude this section by commenting on the economic intuition behind the graph structures giving rise to each inefficiency. When diverse teams are sufficiently productive ($\rho_\ell < \rho^*$), stratification inefficiency arises precisely when it is possible to form two diverse teams, but there exists a diverse team whose formation would cause excluded workers to form a homogeneous team. The only correlation graphs possessing this property are those isomorphic to either Figure 1e, 1g, or 1i. Stable and inefficient matchings exist in such graphs because the outside option for workers in inefficient, diverse teams can never exceed their match value (which is maximized in any equilibrium within a diverse team when $\rho_\ell < \rho^*$).

When diverse teams are less productive ($\rho_\ell \geq \rho^*$), asymmetric effort inefficiency arises whenever it is feasible to form two homogeneous teams and select equilibria within these teams such that the high producers cannot form a diverse team. The correlation graphs possessing this property are those isomorphic to either Figure 1a, 1b, 1c, 1e, or 1g. Stable and inefficient matchings exist in such graphs because the low producers in the homogeneous teams obtain a higher utility than they can attain in any equilibrium in a diverse team (by $\rho_\ell \geq \rho^*$) and the high producers cannot form a deviating diverse team.

The only other correlation graphs in which a stable and asymmetric effort inefficient allocation exist are those isomorphic to Figure 1i (e.g., Figure 5a). In these cases, there is an asymmetric effort inefficient core allocation in which the *only* feasible homogeneous team forms. Such an allocation is stable for a subtly different reason — the high producer in such a team cannot form a profitable deviating team with the other workers because they together compose a diverse team. Hence, they would only benefit from membership in a homogeneous team if the deviating high producer remained a high producer. We remark here that, counter-intuitively, all stable allocations become efficient when we *remove* the edge $(3,4)$ in Figure 5a.

5 Discussion

Our paper is a first step towards understanding how research teams form absent a central authority and in the absence of transfers. We shed light on how workers' incentives



(a) Correlation Graph G_ℓ.

(b) Sorting when $\rho_\ell \geq \rho^*$.

Figure 5: Asymmetric Effort Inefficiency in an Almost Connected Graph.

for effort within teams are affected by their informational complementarities and therefore impact equilibrium sorting. Our analysis uncovers two plausible forces leading to inefficient sorting. First, workers producing complementary information may match and force excluded workers to form highly unproductive teams composed of workers producing substitutable information. Hence, there is too much inequality in productivity *across* teams. Second, even when it is efficient for a team composed of workers producing complementary information to form, such a team may not arise in equilibrium if one of its members has an opportunity to form a less productive team in which she exerts relatively less effort. Hence, there is too much inequality in effort *within* teams. Our theoretical results link the productivity of diverse teams and the network structure of an organization, on one hand, to the efficiency of endogenous sorting, on the other.

Our paper makes several simplifying assumptions. Most starkly, (i) the signal-acquisition game played by workers within a team corresponds to the limit of a sequence of games in which workers acquire a discrete number of signals and (ii) the entries of the correlation matrix among workers can only take on two values. These assumptions enable us to provide the sharpest possible characterization of the sources of sorting inefficiency. Nevertheless, a previous working paper (Kambhampati, Segura-Rodriguez and Shao, 2021) shows that the nature of within-team equilibria and the sources of sorting inefficiency are qualitatively similar when both assumptions are relaxed.

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A Proofs

A.1 Posterior Variance Simplification

For any measurable function $g : X \to \mathbb{R}$, where X is the set of possible realizations of signals,

$$-\mathbb{E}_{x,\theta}\Big[(g(x)-\theta)^2\Big] \leq -\mathbb{E}_x\Big[(\mathbb{E}(\theta \mid x)-\theta)^2\Big] = -\mathbb{E}_x\Big[\mathbb{E}_\theta\Big[(\mathbb{E}(\theta \mid x)-\theta)^2 \mid x\Big]\Big] = -Var(\theta \mid x).$$

The inequality follows because $\mathbb{E}\Big[(b-\theta)^2|x\Big]$ is minimized by setting $b = \mathbb{E}[\theta|x]$. The first equality follows from the Law of Iterated Expectations. The second equality follows from the definition of conditional variance.

Let Σ be the correlation matrix of joint signals x, and 1_N be a N-column vector of ones. The likelihood function of the signals is

$$p(x|\theta) = \det(2\pi K\sigma^{-2}\Sigma)^{-\frac{1}{2}} \exp\bigg(-\frac{1}{2}\Big[(\theta \cdot 1_N - x)' K\sigma^{-2}\Sigma^{-1}(\theta \cdot 1_N - x)\Big]\bigg)$$

and the prior density is

$$p(\theta) = (2\pi)^{-\frac{1}{2}} \exp\bigg(-\frac{1}{2}\Big[\theta^2\Big]\bigg),$$

because θ follows a standard normal distribution. By Bayes rule, the posterior distribution of $\theta|x$ is proportional to

$$\begin{aligned}
p(x|\theta)p(\theta) &\propto \exp\bigg(-\frac{1}{2}\Big[\theta^2 + (\theta \cdot 1_N - x)' K\sigma^{-2}\Sigma^{-1}(\theta \cdot 1_N - x)\Big]\bigg) \\
&\propto \exp\bigg(-\frac{1}{2}\Big[\theta^2(1 + K\sigma^{-2}1_N'\Sigma^{-1}1_N) - \theta K\sigma^{-2}(x'\Sigma^{-1}1_N + 1_N'\Sigma^{-1}x)\Big]\bigg) \\
&\propto \exp\bigg(-\frac{1}{2}\Big[\theta - A\Big]' B\Big[\theta - A\Big]\bigg),
\end{aligned}$$

where $B = (1 + K\sigma^{-2}1_N'\Sigma^{-1}1_N)$, $A = B^{-1}K\sigma^{-2}1_N'\Sigma^{-1}x$, and the proportionality operator eliminates positive constants. Because the derived expression is the kernel of a normal distribution, $Var(\theta \mid x) = B^{-1}$.

We construct B^{-1} when workers take $n_j \geq n_i$ draws. The prior covariance matrix, Σ^{-1}, is block diagonal with n_i blocks of the form

$$\Sigma_0 = \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix},$$

and $n_j - n_i$ scalar blocks each equal to 1. The inverse of a block diagonal matrix is equal to the block diagonal matrix formed by inverting each block. Then, $1_N' \Sigma^{-1} 1_N$ is equal to $n_i 1_2' \Sigma_0^{-1} 1_2 + (n_j - n_i)$. Because

$$\Sigma_0^{-1} = \frac{1}{1-\rho^2} \begin{pmatrix} 1 & -\rho \\ -\rho & 1 \end{pmatrix},$$

we have, $1_2' \Sigma_0^{-1} 1_2 = \frac{2}{1+\rho}$. Hence,

$$\begin{aligned} Var(\theta \mid n_i, n_j) &= B^{-1} \\ &= \left(K\sigma^{-2} 1_N' \Sigma^{-1} 1_N + 1\right)^{-1} \\ &= \left(K\sigma^{-2}\left(\frac{2n_i}{1+\rho} + (n_j - n_i)\right) + 1\right)^{-1} \\ &= \frac{\sigma^2}{K}\left(\frac{n_i(1-\rho)}{1+\rho} + n_j + \frac{\sigma^2}{K}\right)^{-1}. \end{aligned}$$

Define $\underline{n}_{ij} = \min\{n_i, n_j\}$ and $\bar{n}_{ij} = \max\{n_i, n_j\}$. Then, worker i's utility function becomes

$$u_i(n_i, n_j; \rho_{ij}) := \frac{1}{2}\left(1 - \frac{\sigma^2}{K}\left(\underline{n}_{ij}\left(\frac{1-\rho_{ij}}{1+\rho_{ij}}\right) + \bar{n}_{ij} + \frac{\sigma^2}{K}\right)^{-1}\right) - \frac{c}{2K} n_i.$$

A.2 Microfoundation for Continuous Signal-Acquisition Game

We first show that, as K grows large, the action space for each worker in the K-discrete game converges to their action space in the continuous game. We then show that the equilibrium strategy profiles converge to the corresponding strategy profiles in the continuous game.

In what follows, let

$$d^*(X, Y) := \max\{\sup_{x \in X} \inf_{y \in Y} d(x, y), \sup_{y \in Y} \inf_{x \in X} d(x, y)\}$$

be the Hausdorff metric and d be the Euclidean metric. The following Lemma demonstrates the convergence of action sets.

Lemma 3. *Let* $S_K := \{0, \frac{1}{K}, \frac{2}{K}, \ldots, M\}$. *Then,* $d^*(S_K, [0, M]) \to 0$ *as* $K \to \infty$.

Proof. Since $S_K \subset [0, M]$, $\inf_{m \in [0,M]} d(s, m) = 0$ for any $s \in S_K$. Consequently,

$$\sup_{s \in S_K} \inf_{m \in [0,M]} d(s, m) = 0.$$

Then $d^*(S_K, [0, M]) = \sup_{m \in [0,M]} \inf_{s \in S_K} d(s, m)$.

Now we want to show $[0, M] \subset \overline{\cup_{K=1} S_K}$. Let $x \in [0, M]$. We can partition $[0, M]$ into the non-overlapping intervals $[0, \frac{1}{K}) \cup [\frac{1}{K}, \frac{2}{K}) \cup ... \cup [\frac{M-1}{K}, M]$. Therefore, x is in one of these intervals. Denote by x_K the initial point of the interval containing x. Then, by construction, $x_K \to x$ as $K \to \infty$. Therefore, $\{x_K\}_{K=1}^{\infty} \subset \overline{\cup_{K=1} S_K}$. Since $\inf_{s \in S_K} d(s, m)$ is a continuous function in m, the extreme value theorem implies there exists a $m_K^* \in [0, M]$, such that $\sup_{m \in [0,M]} \inf_{s \in S_K} d(s, m) = \inf_{s \in S_K} d(s, m_K^*)$. And since $[0, M] \subset \overline{\cup_{K=1} S_K}$, we have $d^*(S_K, [0, M]) = \inf_{s \in S_K} d(s, m_K^*) \to 0$ as $K \to \infty$. □

The following Lemma demonstrates the convergence of equilibrium strategy profiles.

Lemma 4. *Let* $G_K(\rho) := \Big\{(a^K, b^K) \in \mathbb{Q}^2 |\ (a^K, b^K)$ *is a Nash Equilibrium in the* K*-th discrete game*$\Big\}$ *and* $G(\rho) := \Big\{(a, b) \in \mathbb{R}^2 |\ (a, b)$ *is a Nash Equilibrium in the continuous game*$\Big\}$. *Then, for any* $\rho \in [-1, 1]$, $d^*(G(\rho), G_K(\rho)) \to 0$ *as* $K \to \infty$.

Proof. First, we show that for every convergent sequence $\{(a^K, b^K)\}_{K=1}^{\infty}$ such that $(a^K, b^K) \in G_K(\rho)$, there exists $(a, b) \in G(\rho)$ such that $(a^K, b^K) \to (a, b)$ as $K \to \infty$. For the sake of contradiction, suppose there exists a convergent sequence $\{(a^K, b^K)\}_{K=1}^{\infty}$ with $(a^K, b^K) \in G_K(\rho)$ that does not converge to any $(a, b) \in G(\rho)$. Since $(a^K, b^K) \in [0, M]^2$, the limit of the sequence $\{(a^K, b^K)\}_{K=1}^{\infty}$ exists and we denote it by (a', b'). By our contradiction assumption, we have that (a', b') it is not a Nash equilibrium. Hence, without loss of generality, in the continuous game, there must exist a profitable deviation $a'' \in [0, M]$ for some player i. Moreover, it is always possible to find a sequence $(a''^K)_{K=1}^{\infty}$ with $a''^K \in S_K$ and such that $a''^K \to a''$. As v_i is continuous, there exists large enough K such that $v_i(a^K, b^K) < v_i(a''^K, b^K)$. That is, for large K, in the K-th game, player i has a profitable deviation. This contradicts (a^K, b^K) being in $G_K(\rho)$, which concludes the argument. Now we apply this first result to prove

$$\sup_{(a^K, b^K) \in G_K(\rho)} \inf_{(a,b) \in G(\rho)} d\Big((a^K, b^K), (a, b)\Big) \to 0$$

as $K \to \infty$. Suppose $\sup_{(a^K,b^K)\in G_K(\rho)} \inf_{(a,b)\in G(\rho)} d\left((a^K,b^K),(a,b)\right) > \kappa > 0$. Then there must exist a sequence, $(a^K,b^K) \in G_K(\rho)$ such that $\inf_{(a,b)\in G(\rho)} d\left((a^K,b^K),(a,b)\right) > \kappa$. By construction of S_K, the sequence (a^K,b^K) is bounded. Therefore, it must have a convergent subsequence, (a^{K_k},b^{K_k}). But our first result implies that the subsequence has a limit in $G(\rho)$. Therefore $\inf_{(a,b)\in G(\rho)} d\left((a^{K_k},b^{K_k}),(a,b)\right) \to 0$, contradicting $\inf_{(a,b)\in G(\rho)} d\left((a^K,b^K),(a,b)\right) > \kappa > 0$.

Second, we show that, for every $(a,b) \in G(\rho)$, there exists a convergent sequence $\{(a^K,b^K)\}_{K=1}^{\infty}$ such that $(a^K,b^K) \in G_K(\rho)$ and $(a^K,b^K) \to (a,b)$ as $K \to \infty$. First, observe that $G_K(\rho)$ is non-empty because each discrete game is a potential game (see Kambhampati, Segura-Rodriguez and Shao (2021) for details). Because the strategy spaces are compact, we can always construct a convergent sequence $\{(a^K,b^K)\}_{K=1}^{\infty}$ such that $(a^K,b^K) \in G_K(\rho)$. In addition, from Lemma 3, we know that if either $\rho > 0$ or $\rho < 0$, $G(\rho)$ is a singleton set consisting of a point (a,b). Hence, by the first part of the proof, the sequence $\{(a^K,b^K)\}_{K=1}^{\infty}$ converges to (a,b).

Now, we consider the more difficult case in which $\rho = 0$. Select some Nash equilibrium $(a,b) \in G(0)$. Under the assumption that $0 < c < \min\{\sigma^{-2},\sigma^2\}$, a necessary condition for (a,b) to be an equilibrium is that the marginal utility for both players at (a,b) is equal to the marginal cost of acquiring additional information:

$$\frac{\sigma^2}{(a+b+\sigma^2)^2} = \frac{c}{2}.$$

Denote by $\underline{x}^K$ the largest element in S_K that is less than or equal to the real number x. We claim that either $(\underline{a}^K,\underline{b}^K)$, $(\underline{a}^K + \frac{1}{K},\underline{b}^K)$, or $(\underline{a}^K + \frac{2}{K},\underline{b}^K)$ is a Nash equilibrium of the K-th discrete game, where the "or" is exclusive. Hence, there exists a sequence of equilibria $\{(a^K,b^K)\}_{K=1}^{\infty}$ that converges to (a,b).

To prove the claim, first consider the strategy profile $(\underline{a}^K,\underline{b}^K)$. Because $\underline{a}^K < a$ and $\underline{b}^K < b$, no worker is better off deviating to a lower number of signals. Worker 1 is not better off choosing $\underline{a}^K + \frac{1}{K}$ if the difference between her payoff with the strategy $(\underline{a}^K + \frac{1}{K},\underline{b}^K)$ and the strategy $(\underline{a}^K,\underline{b}^K)$ is negative. That is, if and only if

$$\frac{1/K\sigma^2}{(\underline{a}^K+\underline{b}^K+\sigma^2)(\underline{a}^K+1/K+\underline{b}^K+\sigma^2)} < \frac{c}{2K} \Leftrightarrow$$
$$(a+b+\sigma^2)^2 < (\underline{a}^K+\underline{b}^K+\sigma^2)(\underline{a}^K+1/K+\underline{b}^K+\sigma^2).$$

Because the utility function for information is concave and worker 2 faces the same incentives as worker 1, if this inequality holds, then the profile $(\underline{a}^K,\underline{b}^K)$ is a Nash equilibrium.

If, instead,

$$(a+b+\sigma^2)^2 > (\underline{a}^K + \underline{b}^K + \sigma^2)(\underline{a}^K + 1/K + \underline{b}^K + \sigma^2),$$

then worker 1 always has an incentive to produce at least $\underline{a}^K + 1/K$ signals when worker 2 produces $\underline{b}^K$ signals. The profile $(\underline{a}^K + 1/K, \underline{b}^K)$ is a Nash equilibrium if worker 1 does not strictly prefer to choose $\underline{a}^K + \frac{2}{K}$. This happens if and only if the difference of her payoff when choosing $(\underline{a}^K + \frac{2}{K}, \underline{a}^K)$ and $(\underline{a}^K + \frac{1}{K}, \underline{b}^K)$ is negative. That is, if and only if,

$$(a+b+\sigma^2)^2 < (\underline{a}^K + 1/K + \underline{b}^K + \sigma^2)(\underline{a}^K + 2/K + \underline{b}^K + \sigma^2).$$

Therefore, if this inequality holds, the profile $(\underline{a}^K + 1/K, \underline{b}^K)$ is a Nash equilibrium. Here, it is important to observe that worker 2 does not have an incentive to deviate either: the payoff difference for worker 2 of producing $\underline{b}^K$ signals instead of $\underline{b}^K - 1/K$ when worker 1 produces $\underline{a}^K + 1/K$ signals is the same as the difference in payoffs for worker 1 producing $\underline{a}^K + 1/K$ signals instead of $\underline{a}^K$ when worker 2 produces $\underline{b}^K$ signals. In addition, the payoff difference for player 2 of producing $\underline{b}^K + 1/K$ signals rather than $\underline{b}^K$ signals when worker 1 produces $\underline{a}^K + 1/K$ signals is the same as the the difference in payoffs for worker 1 of producing $\underline{a}^K + 2/K$ signals rather than $\underline{a}^K + 1/K$ when worker 2 produces $\underline{b}^K$ signals.

Finally, if the inequality

$$(a+b+\sigma^2)^2 < (\underline{a}^K + 1/K + \underline{b}^K + \sigma^2)(\underline{a}^K + 2/K + \underline{b}^K + \sigma^2)$$

is not satisfied then worker 1 has an incentive to produce $\underline{a}^K + 2/K$ signals when worker 2 produces $\underline{b}^K$ signals. Worker 1 never wants to deviate to a larger number of signals because the difference of her payoff with the strategy $(\underline{a}^K + \frac{2}{K}, \underline{b}^K)$ and under $(\underline{a}^K + \frac{3}{K}, \underline{b}^K)$ is always negative; the inequality

$$(a+b+\sigma^2)^2 < (\underline{a}^K + 2/K + \underline{b}^K + \sigma^2)(\underline{a}^K + 3/K + \underline{b}^K + \sigma^2)$$

is always satisfied. In an analogous way to the previous, it can be argued that worker 2 has no incentive to deviate from $\underline{b}^K$. We have thus proved the desired claim that either $(\underline{a}^K, \underline{b}^K)$, $(\underline{a}^K + \frac{1}{K}, \underline{b}^K)$, or $(\underline{a}^K + \frac{2}{K}, \underline{b}^K)$ is a Nash equilibrium of the K-th discrete game.

Now we apply this second result to prove

$$\sup_{(a,b)\in G(\rho)} \inf_{(a^K,b^K)\in G_K(\rho)} d\left((a,b),(a^K,b^K)\right) \to 0$$

as $K \to \infty$. Again, $\inf_{(a,b)\in G_K(\rho)} d\left((a,b),(a^K,b^K)\right)$ is continuous in (a,b) and $G(\rho)$ is bounded by construction. By the extreme value theorem, there exists a $(a_K^*, b_K^*) \in G(\rho)$ such that

$\sup_{(a,b)\in G(\rho)} \inf_{(a^K,b^K)\in G_K(\rho)} d\left((a,b),(a^K,b^K)\right) = \inf_{(a^K,b^K)\in G_K(\rho)} d\left((a^*_K,b^*_K),(a^K,b^K)\right)$. But our second result establishes that $\inf_{(a^K,b^K)\in G_K(\rho)} d\left((a^*_K,b^*_K),(a^K,b^K)\right) \to 0$ as $K \to \infty$. $\square$

A.3 Proof of Proposition 1

1. Fix an arbitrary signal acquisition profile with $n_i \geq n_j > 0$, which yields a total surplus of

$$\left(1-\sigma^2(n_j\frac{1-\rho}{1+\rho}+n_i+\sigma^2)^{-1}\right)-\frac{1}{2}cn_i-\frac{1}{2}cn_j.$$

Now, consider an alternative profile with $n_i^* = n_i + n_j$ and $n_j^* = 0$. The profile yields total surplus

$$\left(1-\sigma^2(n_i+n_j+\sigma^2)^{-1}\right)-\frac{1}{2}cn_i-\frac{1}{2}cn_j.$$

Since $\rho > 0$, we have $n_j(\frac{1-\rho}{1+\rho})+n_i < n_i+n_j$. Therefore, the profile (n_i^*, n_j^*) generates a strictly larger total surplus. Since n_i and n_j were arbitrary, we have shown that surplus is maximized only when one of the workers acquires a strictly positive number of signals and the other acquires zero signals.

Now, set $n_j = 0$ so that total surplus is

$$\left(1-\sigma^2(n_i+\sigma^2)^{-1}\right)-\frac{1}{2}cn_i.$$

Any value of $n_i \in \mathbb{R}$ that maximizes this expression satisfies the first-order condition for optimality,

$$\frac{\sigma^2}{(n_i+\sigma^2)^2}-\frac{1}{2}c=0 \quad \Rightarrow \quad n_i=\sqrt{\frac{2\sigma^2}{c}}-\sigma^2.$$

The first-order condition is also sufficient for optimality on $[0,\infty)$ because the surplus function is concave when $n_i \in [0,\infty)$; its second derivative with respect to n_i is

$$\frac{-2\sigma^2}{(n_i+\sigma^2)^3}<0.$$

Because $\sqrt{\frac{2\sigma^2}{c}}-\sigma^2>0$ when $c<\sigma^{-2}$, it follows that $n_j=0$ and $n_i=\sqrt{\frac{2\sigma^2}{c}}-\sigma^2$ maximize total surplus. Analogously, $n_i=0$ and $n_j=\sqrt{\frac{2\sigma^2}{c}}-\sigma^2$ is another maximizer. The efficient total surplus value is obtained by plugging in any of the two solutions into the value function.

2. When $\rho = 0$, total surplus is

$$\left(1 - \sigma^2(q + \sigma^2)^{-1}\right) - cq,$$

where $q = n_i + n_j$. The first-order condition is again necessary and sufficient for optimality:

$$\frac{\sigma^2}{(q + \sigma^2)^2} - \frac{1}{2}c = 0 \quad \Rightarrow \quad q = \sqrt{\frac{2\sigma^2}{c}} - \sigma^2.$$

Thus, any profile satisfying $n_i + n_j = \sqrt{\frac{2\sigma^2}{c}} - \sigma^2$ maximizes total surplus. Again, efficient total surplus is obtained from substitution.

3. Fix any signal acquisition profile with $n_j > n_i$, which yields total surplus

$$\left(1 - \sigma^2(n_i \frac{1-\rho}{1+\rho} + n_j + \sigma^2)^{-1}\right) - \frac{1}{2}cn_i - \frac{1}{2}cn_j.$$

Consider the alternative profile $n_i^* = n_j^* = \frac{n_i + n_j}{2}$, which yields total surplus

$$\left(1 - \sigma^2(n_i \frac{1-\rho}{1+\rho} + \frac{\rho n_i + n_j}{1+\rho} + \sigma^2)^{-1}\right) - \frac{1}{2}cn_i - \frac{1}{2}cn_j.$$

Because $\rho < 0$ and $n_j > n_i$, we have $\frac{\rho n_i + n_j}{1+\rho} > n_j$. Therefore, the profile (n_i^*, n_j^*) generates a higher surplus value. We have thus shown that in any profile that maximizes total surplus, both workers must acquire the same number of signals.

Setting $n_i = n_j = q$, the total surplus function becomes

$$\left(1 - \sigma^2(\frac{2}{1+\rho}q + \sigma^2\right)^{-1}) - cq.$$

The first-order condition is again necessary and sufficient for optimality:

$$-c - \frac{2\sigma^2}{(1+\rho)(\sigma^2 + \frac{2q}{1+\rho})^2} = 0.$$

Solving for q, we obtain that $n_i = n_j = q = (\frac{1+\rho}{2})\left(\sqrt{\frac{1}{1+\rho}}\sqrt{\frac{2\sigma^2}{c}} - \sigma^2\right)$ in any efficient signal profile. Again, efficient total surplus is obtained by substitution.

A.4 Proof of Proposition 2

We first argue that a matching is TU-efficient if and only if it is maximally diverse. Note that the difference in surplus between a diverse team and a homogeneous team,

$$\sqrt{2c\sigma^2} - \sqrt{2c\sigma^2(1+\rho_\ell)} - \frac{1}{2}c\rho_\ell\sigma^2,$$

is strictly positive. To see why, observe that the left-hand side equals zero if ρ_ℓ is set to zero and is strictly decreasing in ρ_ℓ:

$$\frac{d}{d\rho}\left[\sqrt{2c\sigma^2} - \sqrt{2c\sigma^2(1+\rho_\ell)} - \frac{1}{2}c\rho_\ell\sigma^2\right] = -\frac{1}{2}\left(c\sigma^2 + \sqrt{\frac{2c\sigma^2}{1+\rho_\ell}}\right) < 0.$$

Hence, it is immediate that a matching is TU-efficient if and only if it is maximally diverse, i.e., it maximizes the total number of diverse matchings.

Now, we argue that if a matching is TU-stable, then it is maximally diverse. Suppose, towards contradiction, that a TU-stable matching μ is not maximally diverse. Then, there exist two workers i and j such that $\rho_{ij} = \rho_\ell$ and either

1. $\rho_{i\mu(i)} = \rho_{j\mu(j)} = \rho_h$, or
2. without loss of generality, $\rho_{j\mu(j)} = \rho_h$, $\rho_{i\mu(i)} = \rho_\ell$, and $\rho_{\mu(i)\mu(j)} = \rho_\ell$.

In any TU-stable matching, it cannot be that $v_i + v_j < V(\rho_\ell)$. Otherwise, condition (S) of TU-stability would be violated. Henceforth, we assume $v_i + v_j \geq V(\rho_\ell)$. If $\rho_{i\mu(i)} = \rho_{j\mu(j)} = \rho_h$, then $v_{\mu(i)} \leq V(\rho_h) - v_i$ and $v_{\mu(j)} \leq V(\rho_h) - v_j$ by (F). But since $V(\rho_h) < V(\rho_\ell)$ and $v_i + v_j \geq V(\rho_\ell)$, we have $v_i + v_j > V(\rho_h)$. Thus, $v_{\mu(i)} + v_{\mu(j)} \leq V(\rho_h) + V(\rho_h) - (v_i + v_j) < V(\rho_h) \leq V(\rho_{\mu(i)\mu(j)})$, which contradicts condition (S) of TU-stability for the pair $(\mu(i), \mu(j))$. If, instead, $\rho_{j\mu(j)} = \rho_h$, $\rho_{i\mu(i)} = \rho_\ell$, and $\rho_{\mu(i)\mu(j)} = \rho_\ell$, then $v_{\mu(i)} \leq V(\rho_\ell) - v_i$ and $v_{\mu(j)} \leq V(\rho_h) - v_j$ by (F). So, $v_{\mu(i)} + v_{\mu(j)} \leq V(\rho_h) + V(\rho_\ell) - (v_i + v_j) \leq V(\rho_h)$. But, we again arrive at a contradiction of condition (S) of TU-stability for the pair $(\mu(i), \mu(j))$. Therefore, any TU-stable matching is maximally diverse.

Finally, we show that, for any maximally diverse matching μm there exists a vector $v \in \mathbb{R}^4_+$ that satisfies equal treatment of equals under which conditions (F) and (S) in the definition of TU-stability are satisfied. If μ is maximally diverse, then either one diverse team is formed or two diverse teams are formed. If two diverse teams are formed, let $v_i = \frac{V(\rho_\ell)}{2}$. It is immediate that this vector satisfies equal treatment of equals and condition (F). It satisfies condition (S) because there is no team with surplus larger than $V(\rho_\ell)$.

Now, suppose the maximally diverse matching forms exactly one diverse team. Then, the graph G_ℓ is isomorphic to 1a, 1b, 1d or 1f. So, it suffices to describe TU-stabilizing transfers in each Figure. It is useful to note that $c < \frac{1}{8}\sigma^{-2}$ implies $V(\rho_h) \geq \frac{1}{2}V(\rho_\ell)$, so that all constructed transfers are positive. Consider first Figure 1a. The unique maximally diverse matching matches worker 1 and worker 2. Notice, workers 1 and 2 are of the same type and workers 3 and 4 are of the same type. The vector $v_1 = v_2 = \frac{V(\rho_\ell)}{2}$ and $v_3 = v_4 = \frac{V(\rho_h)}{2}$ satisfies equal treatment of equals and satisfies (S) and (F). In Figure 1b, only workers 2 and 3 are of the same type. Any maximally diverse matching is supported as TU-stable matching by the equal treatment of equals vector satisfying $v_1 = v_2 = v_3 = \frac{V(\rho_\ell)}{2}$ and $v_4 = V(\rho_h) - \frac{V(\rho_\ell)}{2}$. In Figure 1d, workers 2, 3 and 4 are of the same type. Then, the equal treatment of equals vector satisfying $v_2 = v_3 = v_4 = \frac{V(\rho_h)}{2}$ and $v_1 = V(\rho_\ell) - \frac{V(\rho_h)}{2}$ supports any maximally diverse matching as TU-stable. Finally, in Figure 1f, workers 1, 2 and 4 are of the same type. The equal treatment of equals vector with $v_1 = v_2 = v_4 = \frac{V(\rho_\ell)}{2}$ and $v_3 = V(\rho_h) - \frac{V(\rho_\ell)}{2}$ satisfies both (S) and (F) under any maximally diverse matching.

A.5 Proof of Proposition 3

1. Suppose first that $\rho > 0$ and consider the profile $\left(\sqrt{\frac{\sigma^2}{c}} - \sigma^2, 0\right)$. In this profile, worker i's marginal value of information is

   $$MV(n_i; n_j, \rho) = \frac{1}{2}c,$$

   which is the marginal cost of acquiring additional information. Hence, worker i is best-responding to n_j. Since $\frac{1-\rho}{1+\rho} \in (0,1)$ when $\rho > 0$, $MV(n_j; n_i, \rho) < MV(n_i; n_j, \rho)$. Hence, the marginal value of information for worker j is strictly less than its cost. So, worker j is best-responding to n_i. It follows that $\left(\sqrt{\frac{\sigma^2}{c}} - \sigma^2, 0\right)$ is a Nash equilibrium. By symmetry of the game, $\left(0, \sqrt{\frac{\sigma^2}{c}} - \sigma^2\right)$ must also be a Nash equilibrium. Substituting these strategy profiles into the payoff functions for each player yields the payoff vectors in the statement of the Proposition.

   To prove that these are the only equilibria, consider any profile (n_i, n_j) with $n_i \geq n_j > 0$. If this profile were to be a Nash equilibrium, then the marginal value of information generated by worker i must be equal to its cost. But then, by $\rho > 0$ and the observation that both workers share the same marginal cost of effort, worker j could reduce n_j and strictly increase her payoff.

2. If $\rho = 0$, then the marginal value of information is identical for both workers given

any signal profile (n_i, n_j). When $n_i + n_j = \sqrt{\frac{\sigma^2}{c}} - \sigma^2$, the marginal value of information equals its marginal cost. So, any such profile constitutes a Nash equilibrium and no other profile can be a Nash equilibrium. The line segment joining the feasible payoff vectors when $\rho > 0$ is thus the set of feasible payoff vectors.

3. If $\rho < 0$, the any profile in which $n_i > n_j$ cannot be a Nash equilibrium. If this profile were to be a Nash equilibrium, then the marginal value of information generated by worker i must be equal to its cost. But, since $\frac{1-\rho}{1+\rho} > 1$ when $\rho < 0$, $MV(n_j; n_i, \rho) > MV(n_i; n_j, \rho)$. So, since both workers share the same marginal cost of effort, worker j would have a strict incentive to acquire more information. It suffices to consider symmetric profiles (n_i, n_j) with $n_i = n_j = n$. The unique value at which the marginal value of information equals the marginal cost, and its corresponding payoff vector, is stated in the Proposition.

A.6 Proof of Corollary 1

1. The unique feasible payoff vector in a diverse team is equal to

$$\frac{1}{2}\left(1 + \frac{c\sigma^2}{2}(1+\rho) - \left(\frac{3-\rho}{2\sqrt{1-\rho}}\right)\sqrt{c\sigma^2(1+\rho)}\right).$$

Its derivative with respect to ρ is

$$\left(\frac{c\sigma^2}{4} - \frac{(3-\rho)c\sigma^2}{4\sqrt{c\sigma^2(1-\rho^2)}}\right) + \left(\frac{\sqrt{c\sigma^2(1+\rho)}}{4\sqrt{1-\rho}} - \frac{3-\rho}{4\sqrt{(1-\rho)^3}}\sqrt{c\sigma^2(1+\rho)}\right).$$

From $c\sigma^2 < \frac{1}{8}$, we have that, for $\rho < 0$, $\frac{(3-\rho)}{\sqrt{c\sigma^2(1-\rho^2)}} > \frac{3}{2}$ and $3 - \rho > 1$. Hence, both bracketed terms are strictly negative and the overall expression is strictly negative.

2. That the payoff vectors are constant in ρ follows directly from Proposition 3. That the low producer obtains a strictly higher utility than the high producer follows from the definition of payoffs and because the assumption that $c\sigma^2 < \frac{1}{8}$ implies that $c\sigma^2 < \sqrt{c\sigma^2}$.

3. Notice that as ρ_ℓ converges to 0, the payoff in a diverse team converges to $\frac{1}{2}\left(1 + \frac{c\sigma^2}{2} - \frac{3}{2}\sqrt{c\sigma^2}\right)$. The payoff of a high producer in a homogeneous team is equal to $\frac{1}{2}(1 + c\sigma^2 - 2\sqrt{c\sigma^2})$. Since $c\sigma^2 < \sqrt{c\sigma^2}$, the former is larger than the latter. Result 1 in this corollary implies that the same inequality is satisfied for $\rho < 0$.

4. As the correlation ρ_ℓ converges to 0, the payoff in a diverse team approaches

$$\frac{1}{2}\left(1+\frac{c\sigma^2}{2}-\frac{3}{2}\sqrt{c\sigma^2}\right).$$

Moreover, the low producer in a homogeneous team obtains $\frac{1}{2}(1-\sqrt{c\sigma^2})$. Because $c\sigma^2 < \sqrt{c\sigma^2}$, when ρ_ℓ is sufficiently small, the low producer obtains a higher payoff than any worker in a diverse team. In the opposite direction, as the correlation ρ_ℓ converges to -1, a worker's payoff in a diverse team approaches $\frac{1}{2}$, which is larger than the payoff of a low producer in a homogeneous team, $\frac{1}{2}(1-\sqrt{c\sigma^2})$. The Intermediate Value Theorem and result 1 of this corollary imply that there is a unique ρ^* at which the worker obtains the same utility in a diverse team as she does as a low producer in a homogeneous team. The worker's payoff is strictly larger as a low producer in a homogeneous team if and only if $\rho_\ell > \rho^*$.

A.7 Proof of Proposition 4

1. It suffices to show that any diverse team generates strictly larger total surplus than any homogeneous team. That is, for any $\rho_h > 0 > \rho_\ell$, the sum of payoffs in team (i,j) is strictly larger if $\rho_{ij} = \rho_\ell$ than if $\rho_{ij} = \rho_h$. From the first property of Proposition 3, the sum of payoffs in a homogeneous team is always

$$1+\frac{1}{2}c\sigma^2-\frac{3}{2}\sqrt{c\sigma^2}.$$

From the third property of Proposition 3 and the first property of Corollary 1, the sum of payoffs in a diverse team is strictly larger than

$$1-\sqrt{c\sigma^2}-\frac{c}{2}\left(\sqrt{\frac{\sigma^2}{c}}-\sigma^2\right)=1+\frac{1}{2}c\sigma^2-\frac{3}{2}\sqrt{c\sigma^2}.$$

The result follows.

2. Because $|\mathcal{M}|$ is finite, there exists a maximally diverse matching and, hence, an efficient matching. To see that any such matching must is stable, observe that, by Corollary 1 part 3, any worker in a diverse team obtains a strictly lower utility as a high producer in a homogeneous team and that any worker in a diverse team can do no better in another diverse team. In addition, any worker who may potentially want to form a deviating homogeneous team with a worker in a diverse team would only have a strict incentive to do so if he was guaranteed to be the low producer.

It follows that no two workers can match and play a Nash equilibrium that makes both strictly better off than under the maximally diverse matching.

A.8 Proof of Lemma 1

By the proof of Corollary 1 part 4, if $\rho_\ell < \rho^*$, in the unique Nash equilibrium in a diverse team, each worker obtains a strictly higher utility than any worker in any equilibrium in a homogeneous team. So, any inefficient and stable allocation is stratification inefficient. Because every correlation graph, G_ℓ, is isomorphic to one of the nine graphs in Figure 1, it suffices to identify which of these graphs possess an inefficient allocation.

We first show that there is no stable and inefficient matching in Figures 1a, 1b, 1c, 1d, 1f, or 1h. We begin with the simple observation that any feasible matching in Figures 1d and 1f is maximally diverse and therefore efficient. Hence, no inefficient allocation exists. In Figures 1a and 1b, any efficient matching forms one diverse team and one homogeneous team. Any inefficient matching forms two homogeneous teams. However, no such matching can be stable; there exist two workers whom can form a diverse team and obtain a strictly higher payoff by Corollary 1 part 3. In Figures 1c and 1h, any efficient matching forms two diverse teams. Moreover, any inefficient and feasible matching forms two homogeneous teams. Hence, again, there exist two workers whom can form a diverse team and obtain a strictly higher payoff by Corollary 1 part 3.

We now exhibit a stable and inefficient matching in Figures 1e, 1g, and 1i. In all three cases, an efficient matching must form two diverse teams. In Figures 1e and 1g, the matching μ with $\mu(1) = 4$ and $\mu(2) = 3$ is inefficient because it forms only one diverse team. However, it is stable. By $\rho_\ell < \rho^*$, worker 1 and worker 4 cannot obtain a strictly higher utility in any other team. Finally, in Figure 1i the matching μ with $\mu(1) = 2$ and $\mu(3) = 4$ is inefficient because it forms only one diverse team. However, it is stable. By $\rho_\ell < \rho^*$, worker 3 and worker 4 cannot obtain a strictly higher utility in any other team.

A.9 Proof of Lemma 2

By the proof of Corollary 1 part 4, if $\rho_\ell > \rho^*$, in the unique Nash equilibrium in a diverse team, each worker obtains a strictly lower utility than a low producer in a homogeneous team. So, any inefficient and stable allocation is asymmetric effort inefficient. Because every correlation graph, G_ℓ, is isomorphic to one of the nine graphs in Figure 1, it suffices to identify which of these graphs possess an inefficient allocation.

We first observe that there is no stable and inefficient matching in Figures 1d, 1f, or 1h. We begin with the simple observation that any feasible matching in Figures 1d and 1f is maximally diverse and therefore efficient. Hence, no inefficient allocation exists. In Figure 1h, any efficient matching forms two diverse teams. Moreover, any inefficient and feasible matching forms two homogeneous teams. For any collection of Nash equilibria within the two homogeneous teams, the two low producers can form a deviating team and obtain a strictly higher payoff by Corollary 1 part 3. Hence, any such matching cannot be stable.

We now exhibit a stable and inefficient allocation in Figures 1a, 1b, 1c, 1e, 1g, and 1i. In Figures 1a, 1b, and 1c, consider the inefficient matching in which $\mu(1) = 4$ and $\mu(2) = 3$. Fix the Nash equilibrium within team $(1,4)$ so that worker 1 is the low producer and worker 4 is the high producer. Fix the Nash equilibrium within team $(2,3)$ so that worker 2 is the low producer and worker 3 is the high producer. Because worker 1 and worker 2 obtain a higher utility than they can in any feasible team in any Nash equilibrium, neither can be a part of any deviating team. Moreover, worker 3 and worker 4 can only form a homogeneous team. Hence, in any Nash equilibrium, one worker does not obtain a strictly higher utility. It follows that the constructed allocation is stable. In Figure 1e, consider the inefficient matching in which $\mu(1) = 2$ and $\mu(3) = 4$. Then, both teams are homogeneous. In team $(1,2)$, fix a Nash equilibrium in which worker 1 is a low producer and worker 2 is a high producer. In team $(3,4)$, fix a Nash equilibrium in which worker 4 is a low producer and worker 3 is a high producer. Because worker 1 and worker 4 obtain a higher utility than they can in any feasible team in any Nash equilibrium, neither can be a part of any deviating team. Moreover, worker 3 and worker 4 can only form a homogeneous team. Hence, in any Nash equilibrium, one worker does not obtain a strictly higher utility. It follows that the constructed allocation is stable. Finally, in Figures 1g and 1i, consider the inefficient matching in which $\mu(1) = 2$ and $\mu(3) = 4$. Then, worker 1 and worker 2 form a homogeneous team. Fix the Nash equilibrium in team $(1,2)$ so that worker 1 is the low producer and worker 2 is the high producer. Then, the residual team $(3,4)$ is diverse. Because worker 1 obtains a higher utility than they can in any feasible team in any Nash equilibrium, she cannot be a part of any deviating team. Moreover, worker 3 and worker 4 can only obtain a higher utility as a low producer in a deviating homogeneous team. Any such equilibrium would not yield worker 2 a strictly higher utility than what she currently obtains. So, the matching is stable.