Online Appendix: "Matching to Produce Information: A Model of Self-Organized Research Teams"

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A Assortative Matching

Chade and Eeckhout (2018) study optimal matching in an information environment related to ours. In theirs, the correlation between signals is constant, but precisions may be heterogeneous. They show that if utilities are transferable and each worker produces only one signal, the reduced form utility obtained from forecasting the state is submodular for a wide range of correlations. Therefore, if teams are composed of two workers, optimal matching is negative assortative: the best worker matches the worst worker, the second best matches the second worst, and so on.

In our environment, workers strategically choose the number of signals they produce and transfers are not possible. Moreover, correlation varies, but precisions are held constant. To isolate the effects of the first two features of our model, we assume in this section that precisions vary, but correlation is held to zero. Our main conclusion is that, perhaps unsurprisingly, it need not be true that the negative assortative matching maximizes welfare, nor that it emerges endogenously.[1]

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[1] That utilities are nontransferable is not necessary to revert their result, but we keep it to preserve the structure of the game we study. Following our approach, the equilibrium of the production game is inherently inefficient due to its public goods nature, while in a fully transferable world this inefficiency disappears. We focus on whether negative assortative matching is optimal given the equilibrium played inside each team.

Suppose each worker produces conditionally independent signals with precisions $\tau_1 < \tau_2 < \ldots < \tau_N$. As in the main text, suppose each agent receives the quadratic loss of her team's optimal forecast and that a team has at most two workers. Then, if workers i and j are in a team together, and produce n_i and n_j signals, the utility loss associated with their forecast is

$$-\frac{1}{\tau_\theta + n_i\tau_i + n_j\tau_j}.$$

An application of Proposition 2 of Chade and Eeckhout (2018) implies that the posterior variance is submodular in $n_i\tau_i$. Consequently, negative assortative matching with respect to $n_i\tau_i$ is optimal when workers are forced to choose one signal.

We consider what happens when i and j are free to choose the number of signals they produce. For simplicity, suppose worker i can produce signals with unit variance, the prior variance is equal to unity, and the cost of drawing n signals is $c(n) = 0.001n^2$. Figure 6 presents the resulting PEN correspondence and shows that, as worker j's signal variance increases, equilibria become asymmetric. Why? Since each of worker j's signals produce less information, fixing n_i and n_j, the marginal value of worker j's last signal decreases. On the other hand, the marginal value of a signal for worker i increases. Both forces lead to asymmetry.

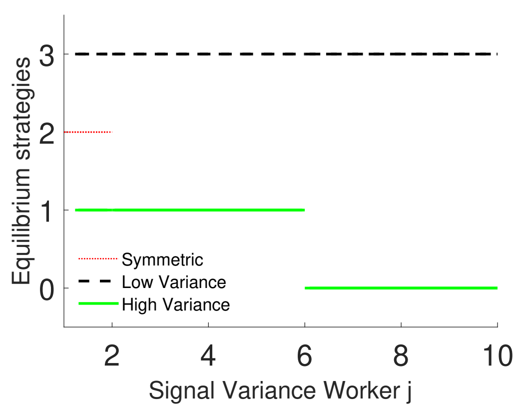

Figure 6: PEN Correspondence when $\rho_{ij} = 0$ and $\tau_i = 1$.

The implications of this behavior for team formation are stark. Suppose that there are four workers with variances 0.25, 0.5, 1 and 1.25. If we match the best worker (the one with variance 0.25) with the worst worker (the one with variance 1.25), the unique PEN played within the team is (2,0); the worst worker does not contribute at all. In contrast, when the worst worker is paired with the worker with variance 1, the unique PEN is $(2, 1)$. Consequently, for small team membership costs, the optimal matching is $\{(0.25, 1), (0.5, 1.25)\}$, instead of the negative assortative matching, $\{(0.25, 1.25), (0.5, 1)\}$. Moreover, it turns out that the optimal matching can be decentralized as a core allocation, while the negative assortative matching cannot.

B PEN Characterization Conditions are Necessary

Consider the equilibrium correspondence presented in Figure 7, where $\sigma^2 = \frac{1}{4} < 1 = \sigma_\theta^2$ violates the sufficient condition for the third and fourth properties in Proposition **??**. In Figure 7, while for $\rho = -0.29$ there is a unique and asymmetric PEN, for a slightly higher correlation there is a unique and symmetric PEN.

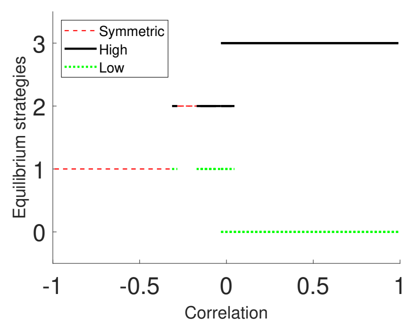

Figure 7: Equilibrium correspondence when $c(m) = 0.019m$, $\sigma^2 = \frac{1}{4}$ and $\sigma_\theta^2 = 1$.

Why does this happen? When $n = 1$, for $\rho = -0.29 \in (\hat{\rho}, \tilde{\rho})$ the marginal value of a signal for a high producer is greater than the marginal value of a signal for a low producer. We may then fix the marginal cost of a second signal so that the high producer wants to produce it. But then, if ρ increases, the marginal value of the low producer *increases* and may exceed the chosen marginal cost, so that she wants to produce a second signal as well. If the low producer produces a second signal, however, the high producer has no incentive to produce a third signal because the information left to learn decreases sufficiently. Hence, a symmetric equilibrium $(2, 2)$ is played.

		Worker i	
		H	L
Worker j	H	$p^2 + \rho_{ij} p(1-p)$	$p(1-p)(1-\rho_{ij})$
	L	$p(1-p)(1-\rho_{ij})$	$(1-p)^2 + \rho_{ij} p(1-p)$

Figure 8: Joint distribution when state is High (H).

C Binary States, Binary Signals

Suppose that the state θ is either High (H) or Low (L). For simplicity, suppose further that $Pr(\theta = H) = \frac{1}{2}$. Each worker can produce an informative signal, with realization H or L realization, and it equals to the true state with probability $p > \frac{1}{2}$. Figure 8 presents the joint distribution over signal realizations when the state is H. If the state is L, the elements of the main diagonal are switched.

Notice that in this environment the feasible set of correlations is bounded below. In particular, statistical feasiblitiy requires that $\rho_{ij} \geq -\frac{1-p}{p}$. Hence, when a couple compares signals and has the most feasible negative correlation they need not learn the state; the state is revealed if HH (or LL) is observed, but not given any other realization. Further, for any correlation, there is a positive probability that HL or LH is observed.

There is no simple expression for the expected posterior variance for an arbitrary profile of signals. Nonetheless, Table 6 computes it for a number of cases; these values are enough to find the PEN of the Production Subgame when each worker's best response is bounded by three. Defining $\tilde{\rho}(t,p)$ and $\hat{\rho}(t,p)$ as in the main text, Figure 9 displays their values when $t = 2$. The figure shows that it is still true that we have $\tilde{\rho}(2,p) > \hat{\rho}(2,p)$ if and only if the precision of the signal is high enough. We suspect a similar result is true for larger t.

D Sequential versus Simultaneous Decision

In this section, we present a finite sequential version of the game played within each team. We assume that the total number of periods $T \geq 2\bar{M}$, where $\bar{M}$ is the upper bound

Table 6: Expected Posterior Variance in the two-state model for some strategies.

# signals i	# signals j	Expected Posterior Variance
0	0	$\frac{1}{4}$
1	0	$p(1-p)$
1	1	$p(1-p)\left(\frac{(p+\rho_{ij}(1-p))(1-p+\rho_{ij}p)}{p^2+(1-p)^2+2\rho_{ij}p(1-p)}+\frac{1}{2}(1-\rho_{ij})\right)$
2	0	$p(1-p)\left(\frac{p(1-p)}{p^2+(1-p)^2}+\frac{1}{2}\right)$
2	1	$p^2(1-p)^2\left(\frac{(p+\rho_{ij}(1-p))(1-p+\rho_{ij}p)}{p^3+(1-p)^3+\rho_{ij}p(1-p)}+2(1-\rho_{ij})+\frac{(p+\rho_{ij}(1-p))(1-p+\rho_{ij}p)}{(1+\rho_{ij})p(1-p)}\right)$
2	2	$p^2(1-p)^2\left(\frac{(p+\rho_{ij}(1-p))^2(1-p+\rho_{ij}p)^2}{(p^2+\rho_{ij}p(1-p))^2+((1-p)^2+\rho_{ij}p(1-p))^2}+\frac{(p+\rho_{ij}(1-p))(1-p+\rho_{ij}p)}{2p(1-p)}+(1-\rho_{ij})^2+\frac{4(1-\rho_{ij})(p+\rho_{ij}(1-p))(1-p+\rho_{ij}p)}{p^2+(1-p)^2+2\rho_{ij}p(1-p)}\right)$
3	0	$p^2(1-p)^2\left(\frac{p(1-p)}{p^3+(1-p)^3}+3\right)$
3	1	$p^2(1-p)^2\left(\frac{p(p-1)(p+\rho_{ij}(1-p))(1-p+\rho_{ij}p)}{p^2(p^2+\rho_{ij}p(1-p))+(1-p)^2((1-p)^2+\rho_{ij}p(1-p))}+(1-\rho_{ij})+\frac{2p(1-p)(1-\rho_{ij})}{p^2+(1-p)^2}+\frac{2(p+\rho_{ij}(1-p))(1-p+\rho_{ij}p)}{p^2+(1-p)^2+2\rho_{ij}p(1-p)}+\frac{(p+\rho_{ij}(1-p))(1-p+\rho_{ij}p)}{p(1-p+\rho_{ij}p)+(1-p)(p+\rho_{ij}(1-p))}\right)$

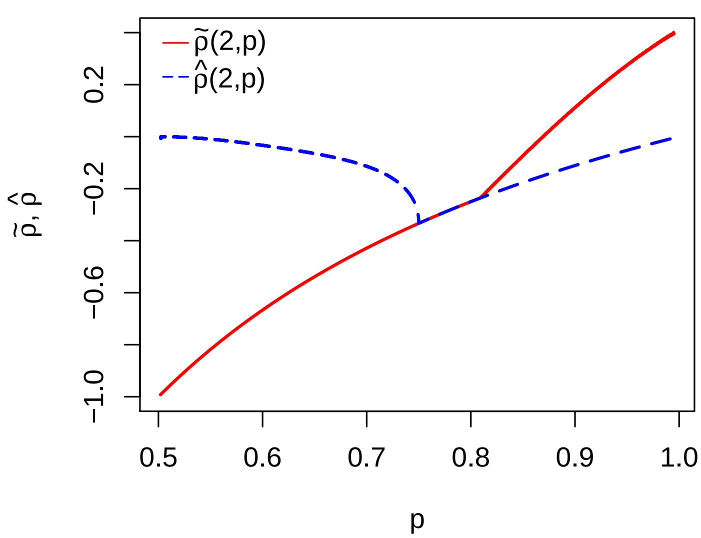

Figure 9: Values $\tilde{\rho}(2,p)$ and $\hat{\rho}(2,p)$ for different signal precisions p.

on best responses described in Lemma **??**. In each period, each worker chooses whether or not to produce a signal $a_t^i \in \{0,1\}$. Signals across periods are conditionally independent and signals in the same period are correlated according to the pairwise correlation of teammates, ρ. In period t, all workers observe all actions a_{t-1} and signals x_{t-1} in periods $1,...t-1$; the public history at period t is given by $h^{t-1} = (a_r, x_r)_{r=1}^{t-1}$ where $a_r = (a_r^1, a_r^2)$.

Let H^{t-1} denote the set of feasible histories up to period t. Then, a strategy for worker i is a function $s_i : \bigcup_{t=1}^{T} H^{t-1} \to \{0,1\}$. The expected payoff of worker i given the history $(a_r, x_r)_{r=1}^{T}$ is:

$$v_i^{(i,j)}(((a_r)_{r=1}^{T})) = -\frac{1}{\left(\frac{2}{1+\rho_{ij}}\sum_{r=1}^{T} a_r^1 a_r^2 + \sum_{r=1}^{T}\left(a_r^1 + a_r^2 - 2a_s^1 a_r^2\right)\right)\sigma^{-2} + \sigma_\theta^{-2}} - c\left(\sum_{r=1}^{T} a_r^i\right).$$

We refer to the equilibrium outcome number of signals as (n_1, n_2), where $n_i = \sum_{r=1}^{T} a_r^i$.

We consider Subgame Perfect Equilibria that are not Pareto Dominated by any other Subgame Perfect Equilibrium– call such an equilibrium a Pareto-Efficient Subgame Perfect Equilibrium (PESP). The next proposition states that, if there is a PEN in the simultaneous game in which strategies differ by at most 1, there is an identical PESP outcome of the sequential game.

Proposition 3 *Let (m_1, m_2) be the most symmetric PEN in the simultaneous game. If $|m_1 - m_2| < 2$, there is a PESP of the sequential game with outcome (n_1, n_2), where $n_1 = m_1$ and $n_2 = m_2$.*

Proof After every history h^{t-1} each worker knows the posterior variance of θ, which we denote by $\sigma^t(h^{t-1})$. We define three automaton states: W_N, W_{D_1}, W_{D_2}. W_N is the state at which no worker deviates, W_{D_1} is the state at which worker 1 is the last deviator, and W_{D_2} is the state at which worker 2 is the last deviator. Consider the strategy profile

$$s_i(h^{t-1}) = \begin{cases} 1 \text{ if } n_i(\sigma^t(h^{t-1})) \geq T - t \\ 0 \text{ otherwise} \end{cases},$$

where $n_i(\sigma^t(h^{t-1}))$ is the number of the most symmetric equilibrium given the prior variance $\sigma^t(h^{t-1})$ and without loss $n_1(\sigma^t(h^{t-1})) \geq n_2(\sigma^t(h^{t-1}))$. Off the path of play choose any Nash equilibrium of the Subgame. If a worker deviates from the prescribed strategy profile then he takes the largest number of signals implied by this Nash equilibrium in the subgame that follows after.

To see why no worker has an incentive to deviate, notice if worker 1 does not produce a signal when she is prescribed to do so, then she can never produce as many signals as she was initially prescribed. But as $|n_1 - n_2| < 2$, worker 2 cannot compensate for worker 1's deviation. As worker 1 prefers to produce n_1 instead of $n_1 - 1$ signals in the simultaneous game, she has no incentive to deviate. A similar argument applies for worker 2. ■

The following example shows why we cannot extend the proposition to all correlations. Suppose $\sigma = \sigma_\theta = 1$ and $c(m) = 0.05m$. If $\rho = 0.15$, the only equilibrium in the simultaneous game is $(3,0)$. However, in the sequential game this cannot be a Subgame Perfect Equilibrium. Suppose worker 1 deviates and decides to produce only one signal in each of the last two periods. Then, the best response of worker 2 is to produce a signal in period $T-1$ or period T. This outcome gives worker 1 a payoff of -0.367 instead of -0.4.[2]

However, for large correlations, the same deviation is not profitable for worker 1 since worker 2 will never want to produce a signal in period T or period $T-1$. If both workers produce a signal during the same period, they would be highly correlated. Hence, worker 2 would not have incentive to produce a signal, since the extra information that is produced by her signal is almost zero. This observation illustrates that, for intermediate correlations, inefficiency due to asymmetric equilibria may be smaller in the extensive game than in the simultaneous game.

Although our intuition suggests that all equilibria of the simultaneous game are more asymmetric than all equilibria of the sequential game, this may not be true. In the following example, there is an asymmetric equilibrium of the sequential game that is more asymmetric than the most symmetric equilibrium of the simultaneous game. Furthermore, it is not an equilibrium of the simultaneous game. Consider the example in Figure 10 in which we graph the equilibrium correspondence of the simultaneous game. For correlation $\rho = 0.1$, the profile $(3,2)$ is the most symmetric equilibrium in the simultane-

[2]In the unique Subgame Perfect Equilibrium, up to identity, worker 1 produces 2 signals and worker 2 produces 1 signal, with no signals taken in the same period.

ous game and $(4, 1)$ is not an equilibrium. However, in the sequential game, the on-path sequence $(a_r)_{r=1}^{T}$, with $a_T^2 = 1$, $a_r^1 = 1$ for $r = T-4, T-3, T-2, T-1$ and $a_r^i = 0$ in any other period, is consistent with a PESP. Notice, all signals are taken in different periods and $(4, 1)$ is the outcome number of signals. A deviation by worker 1 at period $T-4$ is not necessarily followed by an increase in the number of signals by worker 2, since an extra signal by her implies acquiring correlated information. It can be shown that a Nash equilibrium of the Subgame following such a deviation is $(3, 1)$. As $(4, 1)$ is preferred by worker 1 to $(3, 1)$, worker 1 does not have the incentive to deviate at $T-4$. A similar argument applies for deviations in other periods.

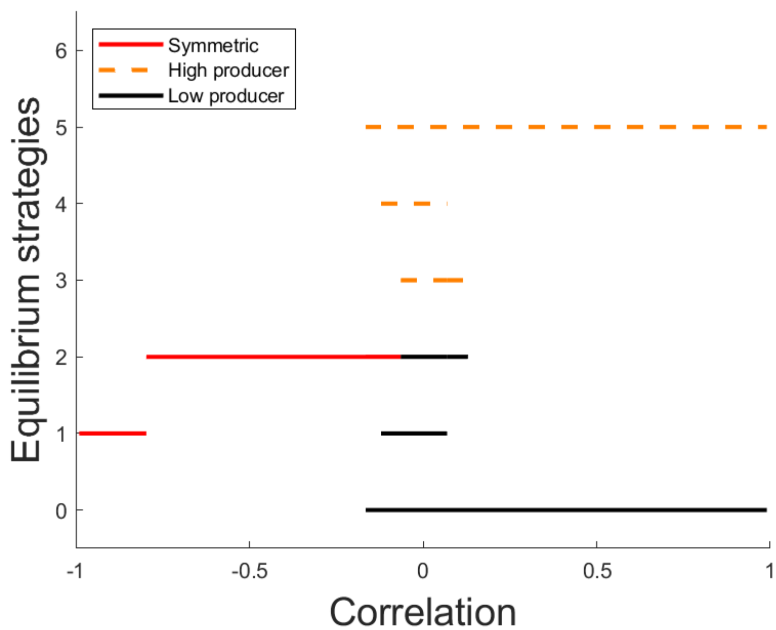

Figure 10: Equilibrium strategies when $c = 0.01n$, $\sigma = \frac{1}{2}$, and $\sigma_\theta = 1$.

E Continuous Action Space

In our model, the informativeness of a signal is scaled by its precision. In this section, we modify the production game by making signals more imprecise and scaling the cost so that there is no "free lunch" effect. This allows us to find a limit game where the action space is continuous.

Let us consider a sequence of games in which each signal becomes less informative. In the kth game, k signals are equivalent to a single signal of the original game. That is, the variance in the kth game, σ_k^2, is equal to $k\sigma^2$, where σ^2 is the variance of each signal in the original game. For simplicity, we assume that the cost of taking a signal is linear. No free lunch implies that in the kth game the cost of a signal is $\frac{c}{k}$, where c is the cost of a signal in the original game. Suppose workers i and j are in a team together and the correlation between their signals is ρ. Then in the kth game, if they choose n_i^k and n_j^k signals, worker i's payoff is given by

$$v_i^{(i,j)}(n_i^k, n_j^k) = \left(\left(\min\left\{ \frac{n_i^k}{k}, \frac{n_j^k}{k} \right\} \frac{2}{1+\rho} + \left| \frac{n_i^k}{k} - \frac{n_j^k}{k} \right| \right) \sigma^{-2} + \sigma_\theta^{-2} \right)^{-1} - c\frac{n_i^k}{k}.$$

Notice that for any real number z and fixed $\epsilon > 0$, there exist rational numbers k and n such $\left|\frac{n}{k} - z\right| < \epsilon$. Therefore, the sequence of games converges to the game where player i chooses $r_i \in \mathbb{R}_+$ and, if workers choose r_i and r_j signals, worker i's payoff is given by

$$v_i^{(i,j)}(r_i, r_j) = \frac{-\sigma^2}{(\underline{r}_{ij}(\phi_{ij} - 1) + \bar{r}_{ij}) + \gamma} - cr_i,$$

where $\underline{r}_{ij} = \min\{r_i, r_j\}$, $\bar{r}_{ij} = \max\{r_i, r_j\}$, $\phi_{ij} = \frac{2}{1+\rho_{ij}}$ and $\gamma = \frac{\sigma^2}{\sigma_\theta^2}$.

As in the discrete game, workers i and j's payoff when in a team together depend on a factor $\phi_{ij} \in [1, \infty)$ that specifies the team's productivity. The equilibrium correspondence is similar to the one described in the main text and characterized in the following proposition.

Proposition 4

- *If $\phi_{ij} < 2$, the unique Nash equilibrium, up to the identity of the workers, is* $\left(0, \sqrt{\frac{\sigma^2}{c}} - \gamma\right)$.
- *If $\phi_{ij} = 2$, any strategy profile such that $r_i + r_j = \sqrt{\frac{\sigma^2}{c}} - \gamma$ is a PEN.*
- *If $\phi_{ij} > 2$, the only PEN is*

$$r_i = r_j = \frac{\sqrt{\frac{\sigma^2(\phi_{ij}-1)}{c}} - \gamma}{\phi_{ij}}.$$

Proof Suppose $r_i > r_j$. Then, the marginal value of r_i for worker i is,

$$\frac{\sigma^2}{\left(r_j(\phi_{ij}-1)+r_i+\gamma\right)^2}$$

and the marginal value of r_j for worker j is,

$$\frac{(\phi_{ij}-1)\sigma^2}{\left(r_j(\phi_{ij}-1)+r_i+\gamma\right)^2}.$$

If $\phi_{ij} < 2$ the marginal value for worker j is always smaller than worker i's marginal value, so there is a corner solution in which $r_j = 0$. Given r_j, i's best-response is $r_i = \sqrt{\frac{\sigma^2}{c}} - \gamma$.

If $\phi_{ij} = 2$, the marginal value of a signal is the same for both workers. Optimally, each chooses r so that the marginal value equals the marginal cost. Since any investment division between the workers does not affect the marginal output, any profile (r_i, r_j) such that $r_i + r_j = \sqrt{\frac{\sigma^2}{c}} - \gamma$ is an equilibrium.

If $\phi_{ij} > 2$, it cannot be the case that $r_i > r_j$ since the marginal benefit for worker j is strictly larger and both workers face the same marginal cost. Hence, all equilibria are symmetric. For (r, r) to be an equilibrium, it must be the case that:

$$\left.\frac{\sigma^2}{\left(r_j(\phi_{ij}-1)+r_i+\gamma\right)^2}\right|_{r_i=r_j} \leq c,$$

and,

$$\left.\frac{(\phi_{ij}-1)\sigma^2}{\left(r_j(\phi_{ij}-1)+r_i+\gamma\right)^2}\right|_{r_i=r_j} \geq c.$$

The only PEN is the profile in which $r = r_i = r_j$ is maximized and satisfies the previous constraints. Hence, the second inequality binds. Re-arranging yields the equation stated in the proposition. ∎

The proposition implies that for negative correlations the only equilibrium is symmetric, for conditionally independent signals there is multiplicity, and for positive correlations the only equilibrium is fully asymmetric.

References

Chade, Hector, and Jan Eeckhout. 2018. "Matching information." *Theoretical Economics*, 13(1): 377–414.