Payoff Continuity in Games with Incomplete Information: An Equivalence Result

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Background

- Equilibrium predictions are sensitive to the common knowledge assumption (Rubinstein (AER-1989)).
- Monderer and Samet (GEB-1989) show that common-p belief is sufficient for payoff continuity of the ϵ-equilibrium correspondence.
- Monderer and Samet (MOR-1996) and Kajii and Morris (JET-1998) show that common p-belief is necessary as well.
- MS define a topology on Partition Models, while KM define a topology on Type Models.

Objective

Establish the sense in which the MS and KM topologies are equivalent.

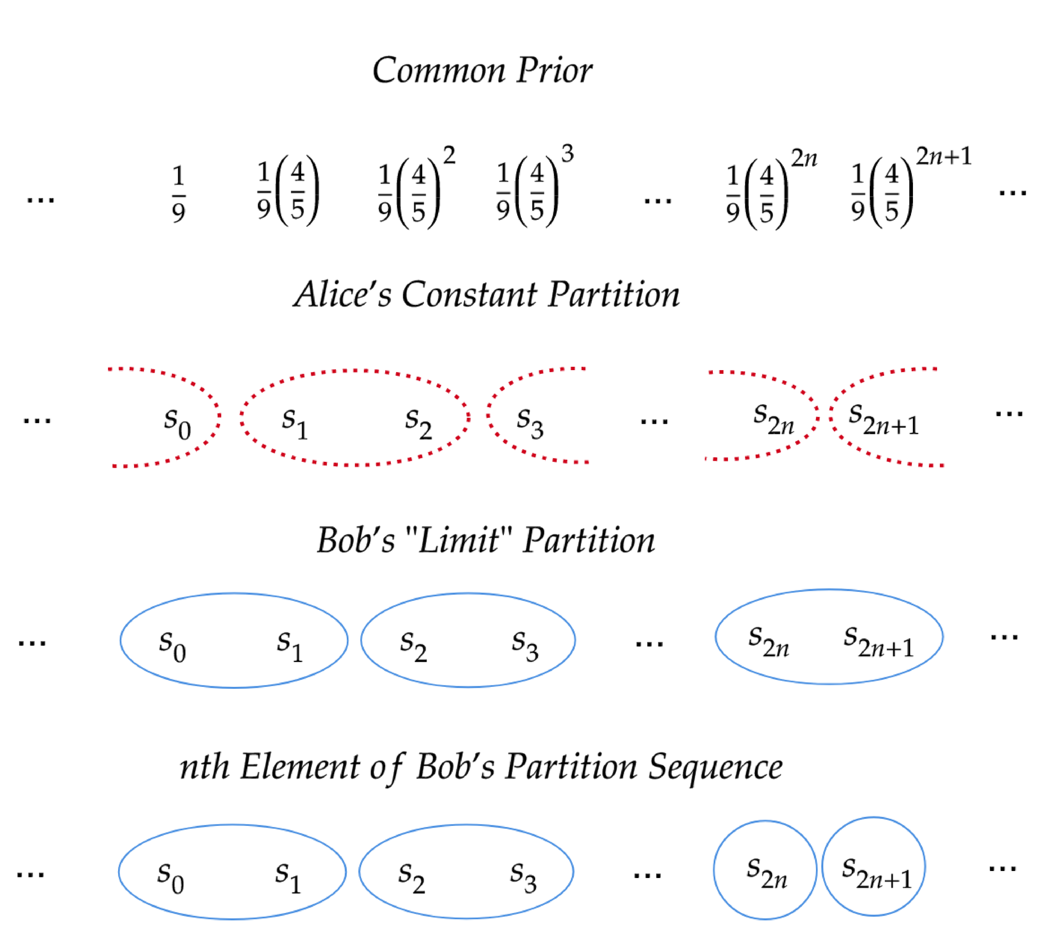
Lower Hemicontinuity Issue

	E	O	C
E	$6,6$	$6,0$	$6,0$
O	$0,6$	$0,0$	$0,0$
C	$0,6$	$0,0$	$4,4$

$\theta = Even.$

	E	O	C
E	$0,0$	$0,6$	$0,0$
O	$6,0$	$6,6$	$6,0$
C	$0,0$	$0,6$	$4,4$

$\theta = Odd.$



Primitives

- Finite set of players $\mathcal{N}$.
- Countable set of states S.
- Countable set of payoff parameters Θ.
- A function $\phi : S \to \Theta$.
- Full support common prior $P \in \Delta(S)$.

Partition Model

- A partition Π_i of S for each player.

Type Model

- A countable set $T := T_1 \times ... \times T_N$.
- A function $\tau : S \to T$.

Canonical Mapping: Partition Model to Type Model

- τ is **Π-consistent** if $\tau_i(s) = \tau_i(s')$ iff $s, s' \in \pi \in \Pi_i$.
- $P \in \Delta(S)$, $\phi : S \to \Theta$, and $\tau : S \to T$ together identify a unique measure $\mu \in \Delta(\Theta \times T)$:

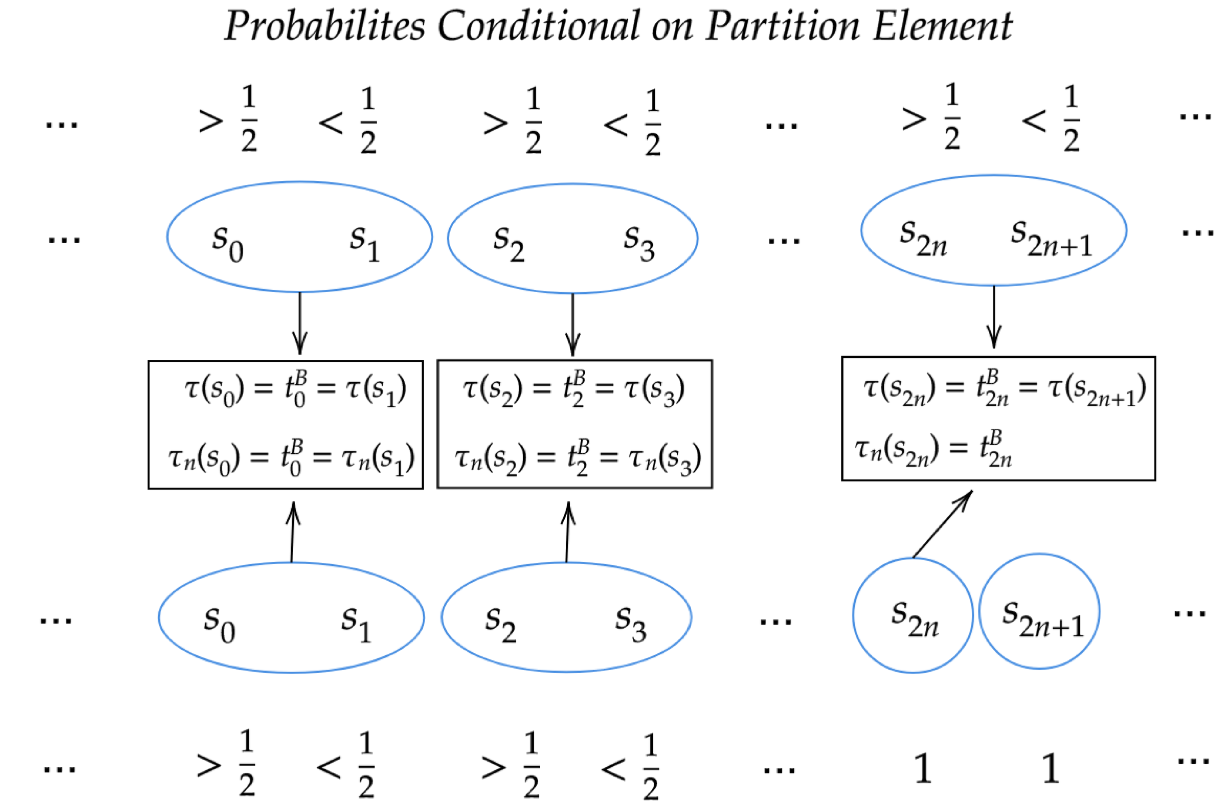
$$\mu(\theta, t) = P(\{s \in S : \phi(s) = \theta \text{ and } \tau(s) = t\}).$$

Labelings: Pairs to Pairs

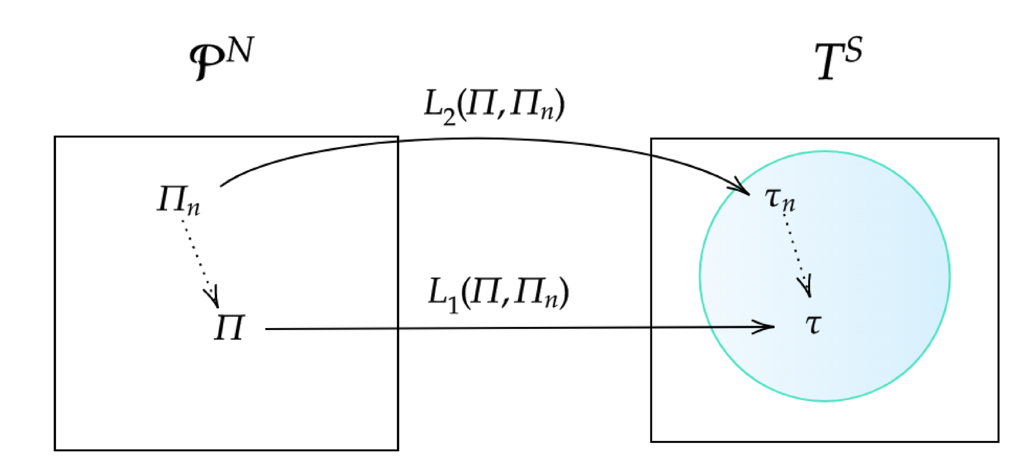
- A **partition labeling** is a function from pairs of Partition Models to pairs of Type Models
$$L : \mathcal{P}^N \times \mathcal{P}^N \to T^S \times T^S.$$
- L is **consistent** if $L(\Pi, \Pi') = (\tau, \tau')$ implies τ is Π-consistent and τ' is Π'-consistent.
- L is **invariant** if for all $\Pi \in \mathcal{P}^N$ there exists a $\tau \in T^S$ such that, $L_1(\Pi, \Pi') = \tau$ for any $\Pi' \in \mathcal{P}^N$.
- L satisfies the **common support condition** if $L(\Pi, \Pi') = (\tau, \tau')$ implies that $\tau(s) = \tau'(s)$ if $s \in I_{\Pi,\Pi'}(1/2)$.

$$d(\pi, \pi') = \max\{P(\pi \backslash \pi' | \pi), P(\pi' \backslash \pi | \pi')\}$$

$$I_{\Pi,\Pi'}(\epsilon) := \cap_{i \in \mathcal{N}} \{s \in S | d(\Pi_i(s), \Pi'_i(s)) < \epsilon\}.$$



Main Result



Suppose L is invariant, consistent, and satisfies the common support condition. If a sequence of Partition Models (Π_n) converges to Π in the MS topology and $L(\Pi, \Pi_n) = (\tau, \tau_n)$ for all n, then the sequence of Type Models (τ_n) converges to τ in the KM topology.

Proof Outline

1. Convert common p-belief statements in Partition Model to common p-belief statements in Type Model. [Use consistency.]
2. Prove that close-by conditional beliefs in MS sense imply close-by conditional beliefs in KM sense. [Use common support condition.]
3. Given that two information structures close in the MS sense are close in the KM sense, use invariance to make limiting statement.

The Converse

- A **type labeling** is a function from pairs of Type Models to pairs of Partition Models
$$\bar{L} : T^S \times T^S \to \mathcal{P}^N \times \mathcal{P}^N.$$
- Define a "consistent" type labeling which coincides with the inverse of any consistent, invariant partition labeling satisfying the common support condition.

Converse Statement: Suppose $\bar{L}$ is "consistent". If a sequence of Type Models (τ_n) converges to τ in the KM topology and $\bar{L}(\tau, \tau_n) = (\Pi, \Pi_n)$ for all n, then the sequence of Type Models (Π_n) converges to Π in the MS topology.