Online Appendices for
"The Optimal Assortativity of Teams Inside the Firm"

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Online Appendix A

Mathematica Code for Figure 1

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(*Primitives and Best-Responses*)
c[e_] := alpha*e^2 + beta*e;
e[q_, bi_] :=
  Piecewise[{{(q*bi)/(2*alpha) - beta/(2*alpha),
     beta <= bi*q <= (alpha + beta)}, {0, bi*q < beta}, {1/2,
     bi*q > (alpha + beta)}}];
S[q_, bi_, bj_] := (e[q, bi] + e[q, bj])*q - c[e[q, bi]] - c[e[q, bj]];
V[q_, bi_, bj_] := (e[q, bi] + e[q, bj])*q*bi - c[e[q, bi]];

(*Objective Function and Constraints*)
profit = muhh*(S[qH, bhh, bhh] - 2*V[qH, bhh, bhh]) +
   mull*(S[qL, bll, bll] - 2*V[qL, bll, bll]) +
   muhl*(S[qM, bhl, blh] - V[qM, bhl, blh] - V[qM, blh, bhl]) +
   muhl*(S[qM, blh, bhl] - V[qM, bhl, blh] - V[qM, blh, bhl]) - (nuh*
      wh + nul*wl);
ICh = (pmuh*V[qH, bhh, bhh] + (1 - pmuh)*V[qM, bhl, blh] +
     wh) - (pmul*V[qH, blh, bhl] + (1 - pmul)*V[qM, bll, bll] + wl);
ICl = (pmul*V[qM, blh, bhl] + (1 - pmul)*V[qL, bll, bll] +
     wl) - (pmuh*V[qM, bhh, bhh] + (1 - pmuh)*V[qL, bhl, blh] + wh);

(*Matching parameters expressed in terms of pmuh and nuh*)
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```
nul = 1 - nuh;
pmul = (nuh/nul)*(1 - pmuh);
muhh = nuh*pmuh;
muhl = nuh*(1 - pmuh);
mulh = muhl;
mull = nul*(1 - pmul);
pNAM = Max[0, (nuh - nul)/nuh];
pPAM = 1;

(*General Parameters*)
alpha = qH;
beta = 1/100*qL;
qH = 90/100;
qL = 10/100;
qM = 49/100*(qH + qL);

(*Figure 1 (a)*)
nuh = 1/10;
data = Module[{x = 0., pts = {}}, While[x <= 50., pmuh = 1 - x/50;
    maxval =
     NMaxValue[{500*profit, ICh >= 0, ICl >= 0, wh >= 0, wl >= 0,
       bhh >= 0, bhl >= 0, blh >= 0,
       bll >= 0}, {{wh, -0.001, 0.001}, {wl, -0.001,
        0.001}, {bhh, -0.001, 0.001}, {bhl, -0.001,
        0.001}, {blh, -0.001, 0.001}, {bll, -0.001, 0.001}}];
    pts = Join[pts, {{pmuh, maxval}}];
    x++]; pts];
ListPlot[data,
 FrameLabel -> {"\!\(\*SubsuperscriptBox[\(p\), \(h\), \(\[Mu]\)]\)",
   "Expected Profit"}, Frame -> True,
 FrameTicks -> {{None, None}, {{0, 1}, None}},
 LabelStyle -> {"Author", Black}, Axes -> False,
 PlotStyle -> Thickness[0.015], Joined -> True]
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(*Figure 1 (b)*)
nuh = 1/100;
data = Module[{x = 0., pts = {}}, While[x <= 50., pmuh = 1 - x/50;
    maxval =
     NMaxValue[{500*profit, ICh >= 0, ICl >= 0, wh >= 0, wl >= 0,
       bhh >= 0, bhl >= 0, blh >= 0,
       bll >= 0}, {{wh, -0.001, 0.001}, {wl, -0.001,
        0.001}, {bhh, -0.001, 0.001}, {bhl, -0.001,
        0.001}, {blh, -0.001, 0.001}, {bll, -0.001, 0.001}}];
    pts = Join[pts, {{pmuh, maxval}}];
    x++]; pts];
ListPlot[data,
 FrameLabel -> {"\!\(\*SubsuperscriptBox[\(p\), \(h\), \(\[Mu]\)]\)",
   "Expected Profit"}, Frame -> True,
 FrameTicks -> {{None, None}, {{0, 1}, None}},
 LabelStyle -> {"Author", Black}, Axes -> False,
 PlotStyle -> Thickness[0.015], Joined -> True]
```

Online Appendix B

Proofs for Section 7, Alternative Timing of Events

Under the alternative timing, the optimal wage scheme implementing PAM, $\mathbf{w} = (b_{hh}, b_{\ell\ell}, w_h, w_\ell) \in \mathbb{R}^4_+$, maximizes

$$\underbrace{\left[\sum_{\theta_i}\sum_{\theta_j}\mu(\theta_i,\theta_j)\cdot\Pi(\theta_i,\theta_j,b_i,b_j)\right]}_{\text{Sum of Expected Profit From Each Team}} - \underbrace{[2\cdot\nu(h)\cdot w_h + 2\cdot\nu(\ell)\cdot w_\ell]}_{\text{Expected Base Wage Payments}}$$

subject to IC_h,

$$V(h,h,b_{hh},b_{hh}) + w_h \geq (e(h,\ell,b_{\ell\ell}) + e(\ell,\ell,b_{\ell\ell}))\cdot q(h,\ell)\cdot b_{\ell\ell} - c(e(h,\ell,b_{\ell\ell})) + w_\ell,$$

and IC_ℓ,

$$V(\ell,\ell,b_{\ell\ell},b_{\ell\ell}) + w_\ell \geq (e(\ell,h,b_{hh}) + e(h,h,b_{hh}))\cdot q(\ell,h)\cdot b_{hh} - c(e(\ell,h,b_{hh})) + w_h.$$

Notice that IC_h and IC_ℓ differ from the constraints analyzed in the main text. In particular, when a worker matches with a teammate of a different type, he continues to exert effort as if he were matched to a teammate of his own type. A worker contemplating misreporting his type thus takes this into account, even though he himself adjusts his effort level. In addition, we remark that an optimal wage scheme continues to exist by the same argument provided in Appendix A of the main text.

In this Online Appendix, we prove that profits under PAM do not converge to those under pure moral hazard in the parameter regions identified in Theorem 4. We need only show that in any solution to the manager's problem, effort distortions for lows do not vanish. The steps of the proof of this claim mirror those in Appendix A. First, we characterize the unique candidate maximizer involving interior effort by both types of workers. In particular, we show that, at this maximizer, both IC_h and IC_ℓ must bind and that $D_{\ell\ell}^{AS} > 0$. Second, we verify that any candidate maximizer must feature interior effort by highs. Hence, the solution to the manager's problem either involves an interior, but distorted, quantity of effort by lows or no effort at all. This concludes the proof.

Candidate Maximizer with Interior Effort

We again verify that the constant rank constraint qualification for the necessity of the KKT optimality conditions holds at any maximizer with interior effort by workers of both types. Let the choice variables be ordered by $(w_h, w_\ell, b_{hh}, b_{\ell\ell})$ and the remaining constraints be ordered by $(LL_h,\ LL_\ell,\ IC_h,\ IC_\ell)$, where LL_θ is the non-negativity constraint on w_θ. Then, the Jacobian with respect to the constraints is given by

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & -1 & q(h,h)\left(2e(h,h,b_{hh}) + \frac{\partial e(h,h,b_{hh})}{\partial b_i} b_{hh}\right) & -q(h,\ell)\left(e(h,\ell,b_{\ell\ell}) + e(\ell,\ell,b_{\ell\ell}) + \frac{\partial e(h,\ell,b_{\ell\ell})}{\partial b_i} b_{\ell\ell}\right) \\ -1 & 1 & -q(h,\ell)\left(e(\ell,h,b_{hh}) + e(h,h,b_{hh}) + \frac{\partial e(h,\ell,b_{hh})}{\partial b_i} b_{hh}\right) & q(\ell,\ell)\left(2e(\ell,\ell,b_{\ell\ell}) + \frac{\partial e(\ell,\ell,b_{\ell\ell})}{\partial b_i} b_{\ell\ell}\right) \end{bmatrix}.$$

The last two rows are linearly independent whenever either $b_{hh} > \frac{\beta}{q(h,h)}$ or $b_{\ell\ell} > \frac{\beta}{q(\ell,\ell)}$, as hold by hypothesis. Hence, the Jacobian has full rank at any interior solution.

Let λ_θ be the Lagrange multiplier on IC_θ and γ_θ be the Lagrange multiplier on the non-negativity constraint of w_θ. The first-order condition with respect to w_h is

$$-2 \cdot \nu(h) + (\lambda_h - \lambda_\ell) + \gamma_h = 0.$$

The first-order condition with respect to w_ℓ is

$$-2 \cdot \nu(\ell) - (\lambda_h - \lambda_\ell) + \gamma_\ell = 0.$$

Summing, we see that

$$\gamma_h + \gamma_\ell = 2.$$

Hence, either $\gamma_h > 0$ or $\gamma_\ell > 0$ (or both). By complementary slackness, it must be that either $w_h = 0$ or $w_\ell = 0$ (or both). The first-order conditions with respect to bonuses result in the following characterization of second-best effort:

$$e(\theta_i, \theta_j, b_{\theta_i\theta_j}) = \phi\left(q(\theta_i, \theta_j)\left[1 + D^{MH}_{\theta_i\theta_j} + D^{AS}_{\theta_i\theta_j}\right]\right),$$

where

$$D^{MH}_{\theta\theta} := -\left[2 \cdot \left(\frac{\partial e(\theta, \theta, b^{MH}_{\theta\theta})}{\partial b_i}\right)^{-1} \cdot e(\theta, \theta, b^{MH}_{\theta\theta}) + b^{MH}_{\theta\theta}\right] \quad \text{and}$$

$$D^{AS}_{\theta\theta} := \frac{\lambda_\theta}{\nu(\theta)}\left[\frac{e(\theta,\theta,b_{\theta\theta})}{\frac{\partial e(\theta,\theta,b_{\theta\theta})}{\partial b_i}} + \frac{b_{\theta\theta}}{2}\right] - \frac{\lambda_{\theta'}}{\nu(\theta)}\left[\frac{q(\theta,\theta')}{q(\theta,\theta)}\left(\frac{e(\theta,\theta',b_{\theta\theta})}{\frac{\partial e(\theta,\theta,b_{\theta\theta})}{\partial b_i}} + \frac{b_{\theta\theta}}{2}\right)\right],$$

with $\theta' \neq \theta$. We use the calculations

$$e(\theta, \theta', b) = \frac{q(\theta,\theta') \cdot b}{2\alpha} - \frac{\beta}{2\alpha} \text{ and}$$
$$\frac{\partial e(\theta, \theta', b)}{\partial b_i} = \frac{q(\theta,\theta')}{2\alpha}$$

to simplify these expressions. In particular, moral hazard distortions are given by

$$D^{MH}_{hh} = -3 \cdot b_{hh} + \frac{2\beta}{q(h,h)} \quad \text{and}$$
$$D^{MH}_{\ell\ell} = -3 \cdot b_{\ell\ell} + \frac{2\beta}{q(\ell,\ell)}.$$

Adverse selection distortions are given by

$$D^{AS}_{hh} = \frac{\lambda_h}{\nu(h)}\left[\frac{3}{2} \cdot b_{hh} - \frac{\beta}{q(h,h)}\right] - \frac{\lambda_\ell}{\nu(h)}\left[\frac{3}{2}\frac{q^2(h,\ell)}{q^2(h,h)} b_{hh} - \beta\frac{q(h,\ell)}{q^2(h,h)}\right] \quad \text{and}$$
$$D^{AS}_{\ell\ell} = \frac{\lambda_\ell}{\nu(\ell)}\left[\frac{3}{2} \cdot b_{\ell\ell} - \frac{\beta}{q(\ell,\ell)}\right] - \frac{\lambda_h}{\nu(\ell)}\left[\frac{3}{2}\frac{q^2(h,\ell)}{q^2(\ell,\ell)} b_{\ell\ell} - \beta\frac{q(h,\ell)}{q^2(\ell,\ell)}\right].$$

Manipulating the first-order conditions with respect to b_{hh} and $b_{\ell\ell}$, we see that

$$b_{hh} = \frac{\nu(h)q^2(h,h) + 2\beta\nu(h)q(h,h) + \lambda_\ell\beta q(h,\ell) - \lambda_h\beta q(h,h)}{4\nu(h)q^2(h,h) + \lambda_\ell\frac{3}{2}q(h,\ell)q(h,h) - \lambda_h\frac{3}{2}q^2(h,h)}$$

and

$$b_{\ell\ell} = \frac{\nu(\ell)q^2(\ell,\ell) + 2\beta\nu(\ell)q(\ell,\ell) - \lambda_\ell\beta q(\ell,\ell) + \lambda_h\beta q(h,\ell)}{4\nu(\ell)q^2(\ell,\ell) - \lambda_\ell\frac{3}{2}q^2(\ell,\ell) + \lambda_h\frac{3}{2}q(h,\ell)q(\ell,\ell)}$$

at any interior solution to the manager's maximization problem.

Proof that both IC_h and IC_ℓ bind.

Suppose $\frac{q(\ell,\ell)}{q(h,h)} \leq M \leq \frac{1}{4}$. We argue that it must be the case that $\lambda_h > 0$ and $\lambda_\ell > 0$ when $q(h,\ell)$ is sufficiently close to $\frac{1}{2}(q(h,h)+q(\ell,\ell))$ by deriving contradictions in the three other cases: (i) $\lambda_\ell = \lambda_h = 0$; (ii) $\lambda_\ell = 0$ and $\lambda_h > 0$; and (iii) $\lambda_\ell > 0$ and $\lambda_h = 0$.

(i) If $\lambda_\ell = \lambda_h = 0$, then $\gamma_\ell > 0$ and $\gamma_h > 0$. Hence, $w_\ell = w_h = 0$ and the first-order conditions imply

$$b_{\theta_i\theta_j} = \frac{1}{4} + \frac{\beta}{2q(\theta_i,\theta_j)} \in (\frac{1}{4}, \frac{1}{2})$$

at any point satisfying the KKT conditions. But, IC_ℓ is satisfied if and only if

$$\frac{3}{4}q(\ell,\ell)^2 b_{\ell\ell}^2 - \beta q(\ell,\ell) b_{\ell\ell} - \frac{1}{4}q(h,\ell)^2 b_{hh}^2 + \frac{\beta}{2}q(h,\ell) b_{hh}$$

$$-\frac{1}{2}q(h,h)q(h,\ell)b_{hh}^2 + \frac{\beta}{2}q(h,h)b_{hh} \geq 0,$$

which is satisfied only if

$$b_{hh} \leq \frac{q(\ell,\ell)}{q(h,\ell)} b_{\ell\ell} := \bar{b}.$$

As $q(h,\ell)$ approaches $\frac{1}{2}(q(h,h)+q(\ell,\ell))$, $\bar{b}$ becomes arbitrarily close to

$$2b_{\ell\ell}\frac{q(\ell,\ell)}{q(h,h)+q(\ell,\ell)}.$$

As $b_{\ell\ell} \leq \frac{1}{2}$ for any parameters at any point satisfying the KKT conditions, and $\frac{q(\ell,\ell)}{q(h,h)+q(\ell,\ell)} < \frac{q(\ell,\ell)}{q(h,h)} \leq \frac{1}{4}$, $\bar{b}$ must eventually lie strictly below $\frac{1}{4}$. Hence, b_{hh} must lie strictly below $\frac{1}{4}$ in order to satisfy IC_ℓ. But, this contradicts our initial expression for b_{hh}, which is always strictly greater than $\frac{1}{4}$.

(ii) If $\lambda_h > 0$ and $\lambda_\ell = 0$, then $\gamma_\ell > 0$ and $w_\ell = 0$. If $w_h = 0$, then, as argued in (i), it must be that

$$b_{hh} \leq \bar{b}$$

in order to satisfy IC_ℓ, where $\bar{b}$ approaches a value strictly smaller than $\frac{1}{4}$ as $q(h,\ell)$ approaches $\frac{1}{2}(q(h,h)+q(\ell,\ell))$. However, by the necessary first-order conditions, it must be that

$$b_{hh} > \frac{1 - \frac{\beta}{q(h,h)}\frac{\lambda_h}{\nu(h)}}{4 - \frac{3}{2}\frac{\lambda_h}{\nu(h)}} > \frac{1}{4}.$$

To see why, notice that the inequality holds if and only if

$$4\frac{\beta}{q(h,h)}\frac{\lambda_h}{\nu(h)} < \frac{3}{2}\frac{\lambda_h}{\nu(h)}.$$

This always holds because

$$4\frac{\beta}{q(h,h)}\frac{\lambda_h}{\nu(h)} < 2\frac{q(\ell,\ell)}{q(h,h)}\frac{\lambda_h}{\nu(h)} < \frac{1}{2}\frac{\lambda_h}{\nu(h)},$$

where the first inequality follows from $\beta < \frac{q(\ell,\ell)}{2}$ and the second from $\frac{q(\ell,\ell)}{q(h,h)+q(\ell,\ell)} < \frac{q(\ell,\ell)}{q(h,h)} \leq \frac{1}{4}$. Hence, we have arrived at our desired contradiction for the case in which $w_h = 0$.

If $w_h > 0$, on the other hand, then $\gamma_h = 0$ and $\lambda_h = 2\nu(h)$. In this case,

$$b_{hh} = 1 \text{ and}$$

$$b_{\ell\ell} = \frac{\nu(\ell)q^2(\ell,\ell) + 2\nu(\ell)\beta q(\ell,\ell) + 2\nu(h)\beta q(h,\ell)}{4\nu(\ell)q^2(\ell,\ell) + 3\nu(h)q(h,\ell)q(\ell,\ell)} < 1.$$

As $w_\ell < w_h$ and $q(\ell,\ell) < q(h,\ell)$, however, IC_ℓ cannot be satisfied.

(iii) If $\lambda_\ell > 0$ (so that IC_ℓ holds with equality) and $\lambda_h = 0$, then $\gamma_h > 0$ and $w_h = 0$. If $w_\ell = 0$, then, as argued in (i), it must be that

$$b_{hh} \leq \bar{b}$$

in order to satisfy IC_ℓ, where $\bar{b}$ approaches a value strictly smaller than $\frac{1}{4}$ as $q(h,\ell)$ approaches $\frac{1}{2}(q(h,h) + q(\ell,\ell))$. However, by the necessary first-order conditions, it must be that

$$b_{hh} = \frac{1 + \frac{2\beta}{q(h,h)} + \lambda_\ell\left(\frac{\beta}{\nu(h)}\frac{q(h,\ell)}{q^2(h,h)}\right)}{4 + \lambda_\ell\left(\frac{3q(h,\ell)}{2\nu(h)q(h,h)}\right)},$$

a decreasing function of λ_ℓ. As the expression is larger than $\frac{1}{4}$ when $\lambda_\ell = 0$, we have arrived at our desired contradiction.

If, on the other hand, $w_\ell > 0$, then $\gamma_\ell = 0$ and $\lambda_\ell = 2\nu(\ell)$. In this case, when $\nu(h)$ is small enough,

$$b_{hh} = \frac{\nu(h)q^2(h,h) + 2\nu(h)\beta q(h,h) + 2\nu(\ell)\beta q(h,\ell)}{4\nu(h)q^2(h,h) + 3\nu(\ell)q(h,h)q(h,\ell)} < \frac{1}{4} \text{ and}$$

$$b_{\ell\ell} = 1.$$

In addition, IC_h requires that

$$3q^2(h,h)b_{hh}^2 - 4\beta b_{hh}q(h,h) - q^2(h,\ell) - 2q(h,\ell)q(\ell,\ell) + 2\beta q(h,\ell) + 2\beta q(\ell,\ell) - 4w_\ell \geq 0.$$

Whenever $q(h,\ell) > \frac{q(h,h)}{2}$, which always holds for $q(h,\ell)$ close to $\frac{1}{2}(q(h,h)+q(\ell,\ell))$, the inequality cannot be satisfied for any $b_{hh} < \frac{1}{4}$.

Proof that the interior maximizer is unique.

Plugging in $\lambda_\ell = 2\nu(\ell) + \lambda_h$, we have

$$b_{hh} = \frac{\nu(h)q^2(h,h) + 2\beta\nu(h)q(h,h) + 2\beta\nu(\ell)q(h,\ell) - \lambda_h\beta(q(h,h) - q(h,\ell))}{4\nu(h)q^2(h,h) + 3\nu(\ell)q^2(h,\ell) - \lambda_h\frac{3}{2}(q^2(h,h) - q(h,h)q(h,\ell))} \tag{B.1}$$

and

$$b_{\ell\ell} = \frac{\nu(\ell)q^2(\ell,\ell) + \lambda_h\beta(q(h,\ell) - q(\ell,\ell))}{\nu(\ell)q^2(\ell,\ell) + \lambda_h\frac{3}{2}(q(h,\ell)q(\ell,\ell) - q^2(\ell,\ell))}.$$

Algebraic manipulation yields $\frac{\partial b_{\ell\ell}}{\partial\lambda_h} < 0$ and $\frac{\partial b_{hh}}{\partial\lambda_h} > 0$. Summing the binding constraints IC_h and IC_ℓ, we see that incentive compatible bonuses, b_{hh} and $b_{\ell\ell}$, must satisfy

$$\frac{1}{4}(3q^2(h,h) - q^2(h,\ell) - 2q(h,\ell)q(h,h))b_{hh}^2 - \frac{\beta}{2}(q(h,h) - q(h,\ell))b_{hh} =$$

$$\frac{1}{4}(q^2(h,\ell) - 3q^2(\ell,\ell) + 2q(h,\ell)q(\ell,\ell))b_{\ell\ell}^2 - \frac{\beta}{2}(q(h,\ell) - q(\ell,\ell))b_{\ell\ell}.$$

From this condition, we conclude that if $q(h,\ell)$ is close to $\frac{q(h,h)+q(\ell,\ell)}{2}$, then (i) $b_{hh} < b_{\ell\ell}$, (ii) the left-hand side is strictly increasing in b_{hh}, and (iii) the right-hand side is strictly increasing in $b_{\ell\ell}$. Hence, as $\frac{\partial b_{\ell\ell}}{\partial\lambda_h} < 0$, the left-hand side is strictly increasing in λ_h and, because $\frac{\partial b_{hh}}{\partial\lambda_h} > 0$, the right-hand side is strictly decreasing in λ_h. Therefore, there exists a unique λ_h at which the equality holds. As the optimal wage scheme is uniquely determined by λ_h, the interior KKT solution is unique.

Proof that $D_{\ell\ell}^{AS} < 0$.

To show that $D_{\ell\ell}^{AS} < 0$, we show that $b_{\ell\ell} < b_{\ell\ell}^{MH} = \frac{1}{4} + \frac{\beta}{2q(\ell,\ell)}$. Let $b_{hh} = \frac{q(h,\ell)+q(\ell,\ell)}{q(h,h)+q(h,\ell)}b_{\ell\ell}$ and denote by λ_h^1 the (unique) multiplier at which the first-order conditions would yield this bonus. With this bonus, the right-hand side of the sum of the incentive compatibility constraints is strictly larger than the left-hand side. Hence, the optimal value of λ_h, λ_h^*, must be larger than λ_h^1.

Denote by λ_h^2 the value of λ_h at which $b_{\ell\ell} = b_{\ell\ell}^{MH}$. As $\nu(h)$ approaches zero and $q(h,\ell)$ approaches

$\frac{1}{2}(q(h,h)+q(\ell,\ell))$, λ_h^2 approaches

$$\frac{\eta}{1-\eta},$$

where $\eta = \frac{q(\ell,\ell)}{q(h,h)}$. We claim that λ_h^1 is larger than λ_h^2. To see this, observe that as $\nu(h)$ and η approach zero and $q(h,\ell)$ approaches $\frac{1}{2}(q(h,h)+q(\ell,\ell))$, when we plug in λ_h^2 into the expression for b_{hh} derived from the first order condition (B.1) we obtain an expression strictly smaller than $\frac{q(h,\ell)+q(\ell,\ell)}{q(h,h)+q(h,\ell)} b_{\ell\ell}^{MH}$. From $\frac{\partial b_{hh}}{\partial \lambda_h} > 0$ and $\frac{\partial b_{\ell\ell}}{\partial \lambda_h} < 0$, we thus conclude that $\lambda_h^* > \lambda_h^1 \geq \lambda_h^2$. Hence, $b_{\ell\ell} < b_{\ell\ell}^{MH}$, as in the main text.

Candidate Maximizers with Non-Interior Effort by Highs

If $b_{hh} \in (0, \frac{\beta}{q(h,h)})$, then the manager could reduce b_{hh} and weakly increase her profits. It thus suffices to show that $b_{hh} = 0$ and $b_{\ell\ell} > \frac{\beta}{q(\ell,\ell)}$ can never be optimal. Observe first that a high masquerading as a low always obtains a higher utility as a low under her wage scheme. As lows obtain strictly positive expected utility from honesty and obedience when $b_{\ell\ell} > \frac{\beta}{q(\ell,\ell)}$, this implies that $w_h > 0$ in order to satisfy IC_h. In addition, any wage scheme in which $w_h > 0$ and $w_\ell > 0$ cannot possibly be optimal; the manager can reduce both wages by a constant and strictly increase her profits. But, if $w_\ell = 0$, then any $w_h > 0$ satisfying IC_h cannot satisfy IC_ℓ. We have arrived at a contradiction.